This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

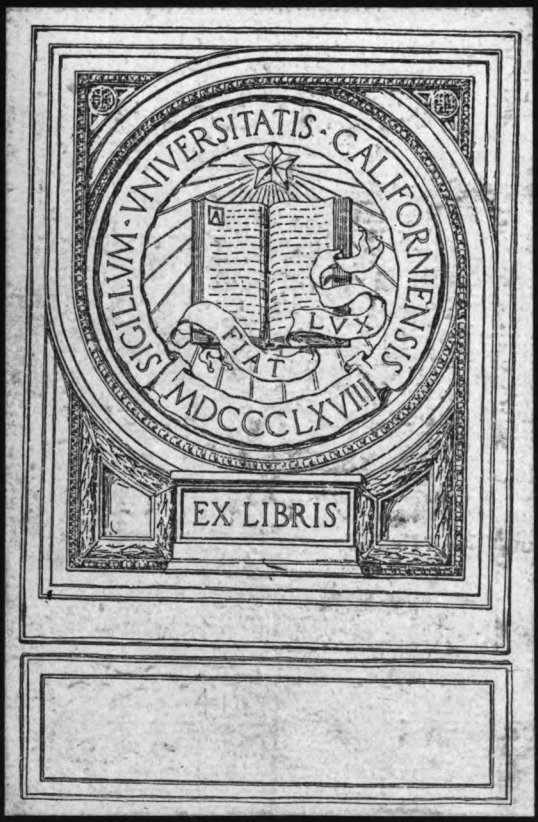
+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

UC-NRLF
$B 259 545


SIGILLVM · VNIVERSITATIS · CALIFORNIENSIS
FIAT LVX
MDCCCLXVIII
EX LIBRIS

Digitized by Google

Digitized by Google

Digitized by Google

Digitized by Google

WORKS OF

PROF. WALTER LORING WEBB

PUBLISHED BY

JOHN WILEY & SONS.

Railroad Construction.—Theory and Practice.

A Text-book for the Use of Students in Colleges and Technical Schools. Fifth Edition. Revised and Enlarged. 16mo. xvii + 789 pages and 217 figures and 10 plates. Morocco, $4.00 net.

Problems in the Use and Adjustment of Engineering Instruments.

Forms for Field-notes; General Instructions for Extended Students' Surveys. 16mo. Morocco, $1.25.

The Economics of Railroad Construction.

Small 8vo. Second Edition. vii + 347 pages, 35 figures. Cloth, $2.50.

The American Civil Engineers' Pocket Book
(*Author of Section on Railroads.*)

Large 16mo. Morocco, $5.00.

Digitized by Google

RAILROAD CONSTRUCTION.

THEORY AND PRACTICE.

A TEXT-BOOK FOR THE USE OF STUDENTS IN COLLEGES AND TECHNICAL SCHOOLS.

BY

WALTER LORING WEBB, C.E.,

Member American Society of Civil Engineers; Member American Railway Engineering Association; Assistant Professor of Civil Engineering (Railroad Engineering) in the University of Pennsylvania, 1893–1901; etc.

FIFTH EDITION, REVISED AND ENLARGED.

TOTAL ISSUE TWELVE THOUSAND.

NEW YORK

JOHN WILEY & SONS, INC.

LONDON: CHAPMAN & HALL, LIMITED

1914

Digitized by Google

TF200
W4
1913

Copyright, 1899, 1903, 1908, 1913,
BY
WALTER LORING WEBB.

Digitized by Google

PREFACE TO FIRST EDITION.

The preparation of this book was begun several years ago, when much of the subject-matter treated was not to be found in print, or was scattered through many books and pamphlets, and was hence unavailable for student use. Portions of the book have already been printed by the mimeograph process or have been used as lecture-notes, and hence have been subjected to the refining process of class-room use.

The author would call special attention to the following features:

a. Transition curves; the multiform-compound-curve method is used, which has been followed by many railroads in this country; the particular curves here developed have the great advantage of being exceedingly simple, and although the method is not theoretically exact, it is demonstrable that the differences are so small that they may safely be neglected.

b. A system of earthwork computations which makes the multiplication and reduction in a single operation by means of a slide-rule and which enables one to compute readily the volume of the most complicated earthwork forms with an accuracy which is only limited by the precision of the cross-sectioning.

c. The "mass curve" in earthwork; the theory and use of this very valuable process.

d. Tables I, II, III, and IV have been computed *ab novo.* Tables I and II were checked (after computation) with other tables, which are generally considered as standard, and all discrepancies were further examined. They are believed to be perfect.

e. Tables V, VI, VII, and IX have been borrowed, by permission, from "Ludlow's Mathematical Tables." It is believed that five-place tables give as accurate results as actual field

308543 Digitized by Google

practice requires. Tables VIII and X have been compiled to conform with Ludlow's system.

The author wishes to acknowledge his indebtedness to Mr. Chas. A. Sims, civil engineer and railroad contractor, for reading and revising the portions relating to the cost of earthwork.

Since the book is written primarily for students of railroad engineering in technical institutions, the author has assumed the usual previous preparation in algebra, geometry, and trigonometry.

WALTER LORING WEBB.

UNIVERSITY OF PENNSYLVANIA,
PHILADELPHIA,
Jan. 1, 1900.

PREFACE TO SECOND EDITION.

SINCE the issue of the first edition the author has conferred with many noted educators in civil engineering, among them the late Professors E. A. Fuertes and J. B. Johnson, regarding the most desirable size of page for this book. The inconvenience of the octavo edition for field-work was found to be limiting its use. It was therefore decided to recast the whole work and reduce the page from "octavo" to "pocket-book" size. Advantage was then taken of the opportunity to revise freely and to add new matter. The original text has now been almost doubled by the addition of several chapters on structures, train resistance, rolling stock, etc., and also several chapters giving the fundamental principles of the economics of railroad location. Those who are familiar with the late Mr. Wellington's masterpiece, "The Economic Theory of Railway Location," will readily appreciate the author's indebtedness to that work. But while the same general method has been followed, the author has taken advantage of the classification of operating expenses adopted by the Interstate Commerce Commission, has used the figures

Digitized by Google

published by them (which were unavailable when Mr. Wellington wrote), and has developed the theory on an independent basis, with the exception of a few minor details. Those who deny the utility of such methods of computation are referred to §§ 367, 426, and elsewhere for a practical discussion of that subject.

The author's primary aim has been to produce a "text-book for students," and the subject-matter has therefore been cut down to that which may properly be required of students in the time usually allotted to railroad work in a civil-engineering curriculum. On this account no extended discussion has been given to the multitudinous forms of various railroad devices in the chapters on structures. The aim has been to teach the principles and to guide the students into proper methods of investigation.

January, 1903.

PREFACE TO THIRD EDITION.

In the present edition Tables IX and X have been entirely changed, the tables having been rewritten so that the values are given for single minutes rather than for each ten minutes. There has also been added a table of squares, cubes, square roots, cube roots, and reciprocals, which are frequently of so great service in computations. Advantage has also been taken of the opportunity to make numerous typographical and verbal changes.

February, 1905.

PREFACE TO FOURTH EDITION.

In this edition a very extensive revision has been made in the chapter on Earthwork. Table XXXIII, giving the volume of level sections, has been added to the book, with a special demonstration of the method of utilizing this table for pre-

Digitized by Google

liminary and approximate earthwork calculations. A demonstration, with table, for determining the economics of ties has also been added. In accordance with the suggestions of Prof. R. B. H. Begg, of Syracuse University, additions have been made to Table IV, which facilitate the solution of problems in transition curves. Very numerous and sometimes extensive alterations and additions, as well as mere verbal and typographical changes, have been made in various parts of the book. The chapters on Economics have been revised to make them conform to more recent estimates of cost of operation.

July, 1908.

PREFACE TO FIFTH EDITION.

THE very radical changes made, during recent years, by the Interstate Commerce Commission, in the analysis of operating expenses of railroads, have required that all of the chapters on Economics should be almost entirely rewritten. The writer has also endeavored to make all recommendations as to construction work to conform with the most recent recommendations on those points by the American Railway Engineering Association, who granted the Author special permission to make such quotations. This gives all such statements the highest possible authority. Advantage was taken of the opportunity to correct minor typographical errors and blemishes which have been discovered since issuing the fourth edition.

March, 1913.

Digitized by Google

TABLE OF CONTENTS.

CHAPTER I.

RAILROAD SURVEYS.

PAGE

RECONNOISSANCE 1

1. Character of a reconnoissance survey. 2. Selection of a general route. 3. Valley route. 4. Cross-country route. 5. Mountain route. 6. Existing maps. 7. Determination of relative elevations. Barometrical method. 8. Horizontal measurements, bearings, etc. 9. Importance of a good reconnoissance.

PRELIMINARY SURVEYS 9

10. Character of a survey. 11. Cross-section method. 12. Cross-sectioning. 13. Stadia method. 14. "First" and "second" preliminary survey.

LOCATION SURVEYS 18

15. "Paper location." 16. Surveying methods. 17. Form of Notes.

CHAPTER II.

ALIGNMENT.

SIMPLE CURVES 19

18. Designation of curves. 19. Length of a subchord. 20. Length of a curve. 21. Elements of a curve. 22. Relation between T, E, and I. 23. Elements of a 1° curve. 24. Exercises. 25. Curve location by deflections. 26. Instrumental work. 27. Curve location by two transits. 28. Curve location by tangential offsets. 29. Curve location by middle ordinates. 30. Curve location by offsets from the long chord. 31. Use and value of the above methods. 32. Obstacles to location. 33. Modifications of location. 34. Limitations in location. 35. Determination of the curvature of existing track. 36. Problems.

COMPOUND CURVES 38

37. Nature and use. 38. Mutual relations of the parts of a compound curve having two branches. 39. Modifications of location. 40. Problems.

Digitized by Google

PAGE

TRANSITION CURVES 43

41. Superelevation of the outer rail on curves. 42. Practical rules for superelevation. 43. Transition from level to inclined track. 44. Fundamental principle of transition curves. 45. Multiform compound curves. 46. Required length of spiral. 47. To find the ordinates of a 1°-per-25-feet spiral. 48. To find the deflections from any point of the spiral. 49. Connection of spiral with circular curve and with tangent. 50. Field-work. 51. To replace a simple curve by a curve with spirals. 52. Application of transition curves to compound curves. 53. To replace a compound curve by a curve with spirals. 53*a*. Use of Table IV.

VERTICAL CURVES 61

54. Necessity for their use. 55. Required length. 56. Form of curve. 57. Numerical example.

CHAPTER III.

EARTHWORK.

FORM OF EXCAVATIONS AND EMBANKMENTS 65

58. Usual form of cross-section in cut and fill. 59. Terminal pyramids and wedges. 60. Slopes. 61. Compound sections. 62. Width of roadbed. 63. Form of subgrade. 64. Ditches. 65. Effect of sodding the slopes, etc.

EARTHWORK SURVEYS 73

66. Relation of actual volume to the numerical result. 67. Prismoids. 68. Cross-sectioning 69. Position of slope-stakes. 70. Setting slope-stakes by means of "automatic" slope-stake rods.

COMPUTATION OF VOLUME 79

71. Prismoidal formula. 72. Averaging end areas. 73. Middle areas. 74. Two-level ground. 75. Level sections. 76. Numerical example, level sections. 77. Equivalent sections. 78. Equivalent level sections. 79. Three-level sections. 80. Computation of products. 81. Five-level sections. 82. Irregular sections. 83. Volume of an irregular prismoid. 84. Numerical example; irregular sections; volume, with approximate prismoidal correction. 85. Magnitude of the probable error of this method. 86 Numerical illustration of the accuracy of the approximate rule. 87. Cross-sectioning irregular sections. 88. Side-hill work. 89. Borrow-pits. 90. Correction for curvature. 91. Eccentricity of the center of gravity. 92. Center of gravity of side-hill sections. 93. Example of curvature correction. 94. Accuracy of earthwork computations. 95. Approximate computations from profiles.

FORMATION OF EMBANKMENTS 114

96. Shrinkage of earthwork. 97. Proper allowance for shrinkage. 98. Methods of forming embankments.

COMPUTATION OF HAUL 120

99. Nature of subject. 100. Mass diagram. 101. Properties

Digitized by Google

PAGE

of the mass curve. 102. Area of the mass curve. 103. Value of the mass diagram. 104. Changing the grade line. 105. Limit of free haul.

ELEMENTS OF THE COST OF EARTHWORK 128

106. Analysis of the total cost into items. 107. Loosening. 108. Loading. 109. Hauling. 110. Choice of method of haul dependent on distance. 111. Spreading. 112. Keeping roadways in order. 112*a*. Trimming cuts to their proper cross-section. 113. Repairs, wear depreciation, and interest on cost of plant. 114. Superintendence and incidentals. 115. Contractor's profit and contingencies. 116. Limit of profitable haul.

BLASTING 149

117. Explosives. 118. Drilling. 119. Position and direction of drill-holes. 120. Amount of explosive. 121. Tamping. 122. Exploding the charge. 123. Cost. 124. Classification of excavated material. 125. Specifications for earthwork.

CHAPTER IV.

TRESTLES.

126. Extent of use. 127. Trestles *vs.* embankments. 128. Two principal types 159

PILE TRESTLES 161

129. Pile bents. 130. Methods of driving piles. 131. Pile-driving formulæ. 132. Pile-points and pile-shoes. 133. Details of design. 134. Cost of pile trestles.

FRAMED TRESTLES 167

135. Typical design. 136. Joints. 137. Multiple-story construction. 138. Span. 139. Foundations. 140. Longitudinal bracing. 141. Lateral bracing. 142. Abutments.

FLOOR SYSTEMS 173

143. Stringers. 144. Corbels. 145. Guard-rails. 146 Ties on trestles. 147. Superelevation of the outer rail on curves. 148. Protection from fire. 149. Timber. 150. Cost of framed timber trestles.

DESIGN OF WOODEN TRESTLES 179

151. Common practice. 152. Required elements of strength. 153. Strength of timber. 154. Loading. 155. Factors of safety. 156. Design of stringers. 157. Design of posts. 158. Design of caps and sills. 159. Bracing.

CHAPTER V.

TUNNELS.

SURVEYING 189

160. Surface surveys. 161. Surveying down a shaft. 162. Underground surveys. 163. Accuracy of tunnel surveying.

DESIGN 194

164. Cross-sections. 165. Grade. 166. Lining. 167. Shafts. 168. Drains.

Digitized by Google

PAGE

CONSTRUCTION..... 199

169. Headings. 170. Enlargement. 171. Distinctive features of various methods of construction. 172. Ventilation during construction. 173. Excavation for the portals. 174. Tunnels *vs.* open cuts. 175. Cost of tunneling.

CHAPTER VI.

CULVERTS AND MINOR BRIDGES.

176. Definition and object. 177. Elements of the design..... 207

AREA OF THE WATERWAY..... 208

178. Elements involved. 179. Methods of computation of area. 180. Empirical formulæ. 181. Value of empirical formulæ. 182. Results based on observation. 183. Degree of accuracy required.

PIPE CULVERTS..... 212

184. Advantages. 185. Construction. 186. Iron-pipe culverts. 187. Tile-pipe culverts.

BOX CULVERTS..... 216

188. Wooden box culverts. 189. Stone box culverts. 190. Old rail culverts. 190*a*. Reinforced concrete culverts.

ARCH CULVERTS..... 221

191. Influence of design on flow. 192. Example of arch-culvert design.

MINOR OPENINGS..... 222

193. Cattle-guards. 194. Cattle-passes. 195. Standard stringer and I-beam bridges.

CHAPTER VII.

BALLAST.

196. Purpose and requirements. 197. Materials. 198. Cross-sections. 199. Methods of laying ballast. 200. Cost..... 227

CHAPTER VIII.

TIES AND OTHER FORMS OF RAIL SUPPORT.

201. Various methods of supporting rails. 202. Economics of ties..... 237

WOODEN TIES..... 238

203. Choice of wood. 204. Durability. 205. Dimensions. 206. Spacing. 207. Specifications. 208. Regulations for laying and renewing ties. 209. Cost of ties.

PRESERVATIVE PROCESSES FOR WOODEN TIES..... 242

210. General principle. 211. Vulcanizing. 212. Creosoting. 213. Burnettizing. 214. Kyanizing. 215. Wellhouse (or zinc-tannin) process. 216. Cost of treating. 217. Economics of treated ties.

Digitized by Google

PAGE

METAL TIES..... 250

218. Extent of use. 219. Durability. 220. Form and dimensions of metal cross-ties. 221. Fastenings. 222. Cost. 223. Bowls or plates. 224. Longitudinals. 224*a*. Reinforced concrete ties.

CHAPTER IX.

RAILS.

225. Early forms. 226. Present standard forms. 227. Weight for various kinds of traffic. 228. Effect of stiffness on traction. 229. Length of rails. 230. Expansion of rails. 231. Rules for allowing for temperature. 232. Chemical composition. 233. Proposed standard specifications for steel rails. 234. Rail wear on tangents. 235. Rail wear on curves. 236. Cost of rails..... 256

CHAPTER X.

RAIL-FASTENINGS.

RAIL-JOINTS..... 270

237. Theoretical requirements for a perfect joint. 238. Efficiency of the ordinary angle-bar. 239. Effect of rail-gap at joints. 240. Supported, suspended, and bridge joints. 241. Failures of rail-joints. 242. Standard angle-bars. 243. Later designs of rail-joints. 243*a*. Proposed specifications for steel splice-bars.

TIE-PLATES..... 276

244. Advantages. 245. Elements of the design. 246. Methods of setting.

SPIKES..... 280

247. Requirements. 248. Driving. 249. Screws and bolts. 250. Wooden spikes.

TRACK-BOLTS AND NUT-LOCKS..... 284

251. Essential requirements. 252. Design of track-bolts. 253. Design of nut-locks.

CHAPTER XI.

SWITCHES AND CROSSINGS.

SWITCH CONSTRUCTION..... 289

254. Essential elements of a switch. 255. Frogs. 256. To find the frog number. 257. Stub switches. 258. Point switches. 259. Switch-stands. 260. Tie-rods. 261. Guard-rails.

MATHEMATICAL DESIGN OF SWITCHES..... 297

262. Design with circular lead rails. 263. Effect of straight frog-rails. 264. Effect of straight point-rails. 265. Combined effect of straight frog-rails and straight point-rails. 266. Comparison of the above methods. 267. Dimensions for a turnout from the OUTER side of a curved track. 268. Dimensions for a turnout from the INNER side of a curved track. 269. Double turnout from a

Digitized by Google

PAGE

straight track. 270. Two turnouts on the same side. 271. Connecting curve from a straight track. 272. Connecting curve from a curved track to the OUTSIDE. 273. Connecting curve from a curved track to the INSIDE. 274. Crossover between two parallel straight tracks. 275. Crossover between two parallel curved tracks. 276. Practical rules for switch-laying.

CROSSINGS 319

277. Two straight tracks. 278. One straight and one curved track. 279. Two curved tracks. 279*a*. Slips.

CHAPTER XII.

MISCELLANEOUS STRUCTURES AND BUILDINGS.

WATER STATIONS AND WATER SUPPLY 326

280. Location. 281. Required qualities of water. 282. Tanks. 283. Pumping. 284. Track tanks. 285. Stand pipes.

BUILDINGS 331

286. Station platforms. 287. Minor stations. 288. Section houses. 289. Engine houses.

SNOW STRUCTURES 336

290. Snow fences. 291. Snow sheds.

TURNTABLES ..... [illegible]

CHAPTER XIII.

YARDS AND TERMINALS.

293. Value of proper design. 294. Divisions of the subject 340

FREIGHT YARDS 341

295. General principles. 296. Relation of yard to main tracks. 297. Minor freight yards. 298. Transfer cranes. 299. Track scales.

ENGINE YARDS 348

300. General principles.

PASSENGER TERMINALS ..... 35[illegible]

CHAPTER XIV.

BLOCK SIGNALING.

GENERAL PRINCIPLES 351

301. Two fundamental systems. 302. Manual systems. 303. Development of the manual system. 304. Permissive blocking. 305. Automatic systems. 306. Distant signals. 307. Advance signals.

MECHANICAL DETAILS ..... 35[illegible]

308. Signals. 309. Wires and pipes. 310. Track circuit for automatic signaling.

Digitized by Google

CHAPTER XV.

ROLLING STOCK.

PAGE

WHEELS AND RAILS 363

311. Effect of rigidly attaching wheels to their axles. 312. Effect of parallel axles. 313. Effect of coning wheels. 314. Effect of flanging locomotive driving wheels. 315. Action of a locomotive pilot-truck.

LOCOMOTIVES.

GENERAL STRUCTURE 370

316. Frame. 317. Boiler. 318. Fire box. 319. Coal consumption. 320. Heating surface. 321. Loss of efficiency of steam pressure. 322. Tractive power.

RUNNING GEAR 381

323. Types of running gear. 324. Equalizing levers. 325. Counterbalancing. 326. Mutual relations of the boiler power, tractive power and cylinder power for various types. 327. Life of locomotives.

CARS.

328. Capacity and size of cars. 329. Stresses to which car-frames are subjected. 330. The use of metal. 331. Draft gear. 332. Gauge of wheels and form of wheel tread 393

TRAIN-BRAKES.

333. Introduction. 334. Laws of friction as applied to this problem 399

MECHANISM OF BRAKES 403

335. Hand-brakes. 336. "Straight" air brakes. 337. Automatic air brakes. 338. Tests to measure the efficiency of brakes. 339. Brake shoes.

CHAPTER XVI.

TRAIN RESISTANCE.

340. Classification of the various forms. 341. Resistances internal to the locomotive. 342. Velocity resistances. 343. Wheel resistances. 344. Grade resistance. 345. Curve resistance. 346. Brake resistances. 347. Inertia resistance. 348. Dynamometer tests. 349. Gravity or "drop" tests. 350. Formulæ for train resistance 409

CHAPTER XVII.

COST OF RAILROADS.

351. General considerations. 352. Preliminary financiering. 353. Surveys and engineering expenses. 354. Land and land

Digitized by Google

PAGE

damages. 355. Clearing and grubbing. 356. Earthwork. 357. Bridges, trestles and culverts. 358. Trackwork. 359. Buildings and miscellaneous structures. 360. Interest on construction. 361. Telegraph lines. 362. Detailed estimate of the cost of a line of road..... 426

PART II.

RAILROAD ECONOMICS.

CHAPTER XVIII.

INTRODUCTION.

363. The magnitude of railroad business. 364. Cost of transportation. 365. Study of railroad economics—its nature and limitations. 366. Outline of the engineer's duties. 367. Justification of such methods of computation..... 436

CHAPTER XIX.

THE PROMOTION OF RAILROAD PROJECTS.

368. Method of formation of railroad corporations. 369. The two classes of financial interests, the security and profits of each. 370. The small margin between profit and loss to the projectors. 371. Extent to which a railroad is a monopoly. 372. Profit resulting from an increase in business done; loss resulting from a decrease. 373. Estimation of probable volume of traffic, and of probable growth. 374. Probable number of trains per day. Increase with growth of traffic. 375. Effect on traffic of an increase in facilities. 376. Loss caused by inconvenient terminals and by stations far removed from business centres. 377. General principles which should govern the expenditure of money for railroad purposes..... 441

CHAPTER XX.

OPERATING EXPENSES.

378. Distribution of gross revenue. 379. Fivefold distribution of operating expenses. 380. Operating expenses per train mile. 381. Reasons for uniformity in expenses per train mile. 382. Detailed classification of expenses with ratios to the total expense. 383. Amounts and percentages of the various items..... 454

Digitized by Google

PAGE

MAINTENANCE OF WAY AND STRUCTURES 463

384. Track material. 385. Roadway and track. 386. Maintenance of track structures. 387. Other items.

MAINTENANCE OF EQUIPMENT 466

388. Superintendence. 389. Repairs, renewals, and depreciation of steam locomotives. 390. Repairs, renewals and depreciation of electric locomotives. 391. Repairs, renewals and depreciation of passenger cars, and of freight cars. 392. Electric equipment; marine equipment; shop machinery and tools; stationery and printing; other expenses. 393. Traffic.

TRANSPORTATION 467

394. Superintendence. 395. Yard engine expenses. 396. Road enginemen. 397. Fuel for road locomotives. 398. Water, lubricants, and other supplies for road locomotives. 399. Road trainmen. 400. Train supplies and expenses. 401. Clearing wrecks, loss, damage and injuries to persons and property. 402. Operating joint tracks and facilities.

CHAPTER XXI.

DISTANCE.

403. Relation of distance to rates and expenses. 404. The conditions other than distance that affect the cost; reasons why rates are usually based on distance 474

EFFECT OF DISTANCE ON OPERATING EXPENSES 475

405. Effect of slight changes in distance on maintenance of way. 406. Effect on maintenance of equipment. 407. Effect on conducting transportation. 408. Estimate of total effect on expenses of small changes in distance (measured in feet); estimate for distances measured in miles.

EFFECT OF DISTANCE ON RECEIPTS 486

409. Classification of traffic. 410. Method of division of through rates between the roads run over. 411. Effect of a change in the length of the home road on its receipts from through competitive traffic. 412. The most advantageous conditions for roads forming part of a through competitive route. 413. Effect of the variations in the length of haul and the classes of the business actually done. 414. General conclusions regarding a change in distance. 415. Justification of decreasing distance to save time. 416. Effect of change of distance on the business done.

CHAPTER XXII.

CURVATURE.

417. General objections to curvature. 418. Financial value of the danger of accident due to curvature. 419. Effect of curvature on travel. 420. Effect on operation of trains 492

Digitized by Google

PAGE

EFFECT OF CURVATURE ON OPERATING EXPENSES..... 496

421. Relation of radius of curvature and of degrees of central angle to operating expenses. 422. Effect of curvature on maintenance of way. 423. Effect of curvature on maintenance of equipment. 424. Effect of curvature on conducting transportation. 425. Estimate of total effect per degree of central angle. 426. Reliability and value of the above estimate.

COMPENSATION FOR CURVATURE..... 504

427. Reasons for compensation. 428. The proper rate of compensation. 429. The limitations of maximum curvature.

CHAPTER XXIII.

GRADE.

430. Two distinct effects of grade. 431. Application to the movement of trains of the laws of accelerated motion. 432. Construction of a virtual profile. 433. Use value and possible misuse. 434. Undulatory grades; advantages, disadvantages, and safe limits..... 509

MINOR GRADES..... 516

435. Basis of cost of minor grades. 436. Classification of minor grades. 437. Effect on operating expenses. 438. Estimate of the cost of one foot of change of elevation. 439. Operating value of the filling of a sag in a grade.

RULING GRADES..... 523

440. Definition. 441. Choice of ruling grades. 442. Maximum train load on any grade. 443. Proportion of traffic affected by the ruling grade. 444. Financial value of increasing the train load. 445. Operating value of a reduction in the rate of the ruling grade.

PUSHER GRADES..... 532

446. General principles underlying the use of pusher engines. 447. Balance of grades for pusher service. 448. Operation of pusher engines. 449. Length of a pusher grade. 450. Cost of pusher engine service. 451. Numerical comparison of pusher and through grades.

BALANCE OF GRADES FOR UNEQUAL TRAFFIC..... 542

452. Nature of the subject. 453. Computation of the theoretical balance. 454. Computation of relative traffic.

CHAPTER XXIV.

THE IMPROVEMENT OF OLD LINES.

455. Classification of improvements. 456. Advantages of re-locations. 457. Disadvantages of re-locations..... 546

REDUCTION OF VIRTUAL GRADE..... 549

458. Obtaining data for computations. 459. Use of the data obtained. 460. Reducing the starting grade at stations.

Digitized by Google

PAGE

APPENDIX. THE ADJUSTMENTS OF INSTRUMENTS 554

TABLES.

I. Radii of curves 564
II. Tangents, external distances, and long chords for a 1° curve. 568
III. Switch leads and distances 571
IV. Transition curves 573
V. Logarithms of numbers 582
VI. Logarithmic sines and tangents of small angles 602
VII. Logarithmic sines, cosines, tangents, and cotangents 605
VIII. Logarithmic versed sines and external secants 650
IX. Natural sines, cosines, tangents, and cotangents 695
X. Natural versed sines and external secants 718
XI. Reduction of barometer reading to 32° F 741
XII. Barometric elevations 742
XIII. Coefficients for corrections for temperature and humidity .. 742
XIV. Capacity of cylindrical water-tanks in United States standard gallons of 231 cubic inches 329
XV. Number of cross ties per mile 430
XVI. Tons per mile (with cost) of rails of various weights 431
XVII. Splice bars and bolts per mile of track 432
XVIII. Railroad spikes 433
XIX. Track bolts 433
XX. Classification of operating expenses of all railroads 470–473
XXI. Effect on operating expenses of changes in distance 485
XXII. Effect on operating expenses of changes in curvature 501
XXIII. Velocity head of trains 512
XXIV. Effect on operating expenses of changes in grade 520
XXV. Tractive power of locomotives 525
XXVI. Total train resistance per ton on various grades 527
XXVII. Cost of an additional train to handle a given traffic 532
XXVIII. Balanced grades for one, two and three engines 536
XXIX. Cost for each mile of pusher engine service 540
XXX. Useful trigonometrical formulæ 743
XXXI. Useful formulæ and constants 745
XXXII. Squares, cubes, square roots, cube roots and reciprocals 746
XXXIII. Cubic yards per 100 feet of level sections 763
XXXIV. Annual charge against a tie, based on the original cost and assumed life of the tie 766

INDEX 767

Digitized by Google

Digitized by Google

UNIV. OF CALIFORNIA

RAILROAD CONSTRUCTION.

CHAPTER I.

RAILROAD SURVEYS.

THE proper conduct of railroad surveys presupposes an adequate knowledge of almost the whole subject of railroad engineering, and particularly of some of the complicated questions of Railroad Economics, which are not generally studied except at the latter part of a course in railroad engineering, if at all. This chapter will therefore be chiefly devoted to methods of instrumental work, and the problem of choosing a general route will be considered only as it is influenced by the topography or by the application of those elementary principles of Railroad Economics which are self-evident or which may be accepted by the student until he has had an opportunity of studying those principles in detail

RECONNOISSANCE SURVEYS.

1. **Character of a reconnoissance survey.** A reconnoissance survey is a very hasty examination of a belt of country to determine which of all possible or suggested routes is the most promising and best worthy of a more detailed survey. It is essentially very rough and rapid. It aims to discover those salient features which instantly stamp one route as distinctly superior to another and so narrow the choice to routes which are so nearly equal in value that a more detailed survey is necessary to decide between them.

2. **Selection of a general route.** The general question of running a railroad between two towns is usually a financial rather

Digitized by Google

than an engineering question. Financial considerations usually determine that a road *must* pass through certain more or less important towns between its termini. When a railroad runs through a thickly settled and very flat country, where, from a topographical standpoint, the road may be run by any desired route, the "right-of-way agent" sometimes has a greater influence in locating the road than the engineer. But such modifications of alignment, on account of business considerations, are foreign to the engineer's side of the subject, and it will be hereafter assumed that topography alone determines the location of the line. The consideration of those larger questions combining finance and engineering (such as passing by a town on account of the necessary introduction of heavy grades in order to reach it) will be taken up in Chap. XIX, *et seq.*

3. **Valley route.** This is perhaps the simplest problem. If the two towns to be connected lie in the same valley, it is frequently only necessary to run a line which shall have a nearly uniform grade. The reconnoissance problem consists largely in determining the difference of elevation of the two termini of this division and the approximate horizontal distance so that the proper grade may be chosen. If there is a large river running through the valley, the road will probably remain on one side or the other throughout the whole distance, and both banks should be examined by the reconnoissance party to determine which is preferable. If the river may be easily bridged, both banks may be alternately used, especially when better alignment is thereby secured. A river valley has usually a steeper slope in the upper part than in the lower part. A uniform grade throughout the valley will therefore require that the road climbs up the side slopes in the lower part of the valley. In case the "ruling grade" * for the whole road is as great as or greater than the steepest natural valley slope, more freedom may be used in adopting that alignment which has the least cost—regardless of grade. The natural slope of large rivers is almost invariably so low that grade has no influence in determining the choice of location. When bridging is necessary, the river banks should be examined for suitable locations for abutments

* The *ruling grade* may here be loosely defined as the maximum grade which is permissible. This definition is not strictly true, as may be seen later when studying Railroad Economics, but it may here serve the purpose.

Digitized by Google

and piers. If the soil is soft and treacherous, much difficulty may be experienced and the choice of route may be largely determined by the difficulty of bridging the river except at certain favorable places.

4. **Cross-country route.** A cross-country route always has one or more summits to be crossed. The problem becomes more complex on account of the greater number of possible solutions and the difficulty of properly weighing the advantages and disadvantages of each. The general aim should be to choose the lowest summits and the highest stream crossings, provided that by so doing the grades between these determining points shall be as low as possible and shall not be greater than the ruling grade of the road. Nearly all railroads combine cross-country and valley routes to some extent. Usually the steepest natural slopes are to be found on the cross-country routes, and also the greatest difficulty in securing a low through grade. An approximate determination of the ruling grade is usually made during the reconnoissance. If the ruling grade has been previously decided on by other considerations, the leading feature of the reconnoissance survey will be the determination of a general route along which it will be possible to survey a line whose maximum grade shall not exceed the ruling grade.

5. **Mountain route.** The streams of a mountainous region frequently have a slope exceeding the desired ruling grade. In such cases there is no possibility of securing the desired grade by following the streams. The penetration of such a region may only be accomplished by "development"—accompanied perhaps by tunneling. "Development" consists in deliberately increasing the length of the road between two extremes of elevation so that the rate of grade shall be as low as desired. The usual method of accomplishing this is to take advantage of some convenient formation of the ground to introduce some lateral deviation. The methods may be somewhat classified as follows:

(a) Running the line up a convenient lateral valley, turning a sharp curve and working back up the opposite slope. As shown in Fig. 1, the considerable rise between A and B was surmounted by starting off in a very different direction from the general direction of the road; then, when about one-half of the desired rise had been obtained, the line crossed the valley and continued the climb along the opposite slope. (b) *Switch-*

Digitized by Google

back. On the steep side-hill *BCD* (Fig. 1) a very considerable gain in elevation was accomplished by the switchback *CD*. The gain in elevation from *B* to *D* is very great. On the other hand, the speed must always be slow; there are two complete stoppages of the train for each run; all trains must run backward from *C* to *D*. (c) *Bridge spiral*. When a valley is so narrow at some point that a bridge or viaduct of reasonable length can span the valley at a considerable elevation above the

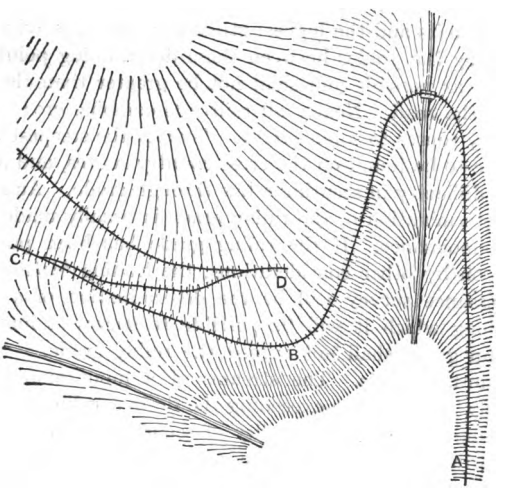


FIG. 1.

bottom of the valley, a bridge spiral may be desirable. In Fig. 2 the line ascends the stream valley past *A*, crosses the stream at *B*, works back to the narrow place at *C*, and there crosses itself, having gained perhaps 100 feet in elevation. (d) *Tunnel spiral* (Fig. 3). This is the reverse of the previous plan. It implies a thin steep ridge, so thin at some place that a tunnel through it will not be excessively long. Switchbacks and spirals are sometimes necessary in mountainous countries, but they should not be considered as normal types of construction. A region must be very difficult if these devices cannot be avoided.

On Plate I are shown three separate ways (as actually constructed) of running a railroad between two points a little over three miles apart and having a difference of elevation of nearly

Digitized by Google

Digitized by Google

Digitized by Google

1100 feet. At *A* the Central R. R. of New Jersey runs *under* the Lehigh Valley R. R. and soon turns off to the northeast for about six miles, then doubles back, reaching *D*, a fall of about 1050 feet with a track distance of about 12.7 miles. The L. V. R. R. at *A* runs to the westward for six to seven miles,

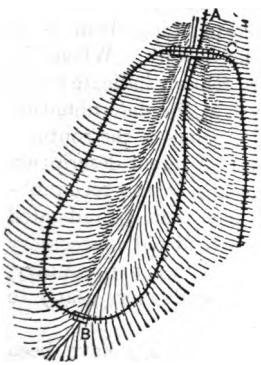


FIG. 2.

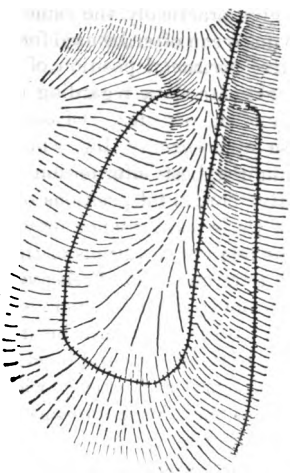


FIG. 3.

then turns back until the roads are again close together at *D*. The track distance is about 14 miles and the drop a little greater, since at *A* the L. V. R. R. crosses *over* the other, while at *D* they are at practically the same level. From *B* to *C* the distance is over eleven miles. From *A* directly down to *D* the C. R. R. of N. J. runs a "gravity" road, used exclusively for freight, on which cars alone are hauled by cable. The main-line routes are remarkable examples of sheer "development." Even as constructed the L. V. R. R. has a grade of about 95 feet per mile, and this grade has proved so excessive for freight work that the company has constructed a cut-off (not shown on the map) which leaves the main line at *A*, nearly parallels the C. R. R. to *C*, and then running in a northeasterly direction again joins the main line beyond Wilkesbarre. The grade is thereby cut down to 65 feet per mile.

Rack railways and cable roads, although types of mountain railroad construction, will not be here considered.

Digitized by Google

6. Existing maps. The maps of the U. S. Geological Survey are exceedingly valuable as far as they have been completed So far as topographical considerations are concerned, they almost dispense with the necessity for the reconnoissance and "first preliminary" surveys. Some of the State Survey maps will give practically the same information. County and township maps can often be used for considerable information as to the relative *horizontal* position of governing points, and even some approximate data regarding elevations may be obtained by a study of the streams. Of course such information will not dispense with surveys, but will assist in so planning them as to obtain the best information with the least work. When the relative horizontal positions of points are reliably indicated on a map, the reconnoissance may be reduced to the determination of the relative elevations of the governing points of the route.

7. Determination of relative elevations. A recent description of European methods includes spirit-leveling in the reconnoissance work. This may be due to the fact that, as indicated above, previous topographical surveys have rendered unnecessary the "exploratory" survey which is required in a new country, and that their reconnoissance really corresponds more nearly to our preliminary.

The perfection to which barometrical methods have been brought has rendered it possible to determine differences of elevation with sufficient accuracy for reconnoissance purposes by the combined use of a mercurial and an aneroid barometer. The mercurial barometer should be kept at "headquarters," and readings should be taken on it at such frequent intervals that any fluctuation is noted, and throughout the period that observations with the aneroid are taken in the field. At each observation there should also be recorded the time, the reading of the attached thermometer, and the temperature of the external air. For uniformity, the mercurial readings should then be "reduced to 32° F." The form of notes for the mercurial barometer readings should be as follows:

Time.	Merc. Barom.	Attached Therm.	Reduction to 32° F.	External Therm.	Corrected reading.
7:00 A. M.	29.872	72°	—.117	73°	29.755
:15	.866	73.5	.121	75	.745
:30	.858	75	.125	76	.733
:45	.850	76	.127	77	.723

Digitized by Google

The corrections in column 4 are derived from Table XI by interpolation.

Before starting out, a reading of the aneroid should be taken at headquarters coincident with a reading of the mercurial. The difference is one value of the correction to the aneroid. As soon as the aneroid is brought back another comparison of readings should be made. Even though there has been considerable rise or fall of pressure in the interval, the *difference* in readings (the correction) should be substantially the same provided the aneroid is a good instrument. If the difference of elevation is excessive (as when climbing a high mountain), even the best aneroid will "lag" and not recover its normal reading for several hours, but this does not apply to such differences of elevation as are met with in railroad work. The best aneroids read directly to $\frac{1}{100}$ of an inch of mercury and may be estimated to $\frac{1}{1000}$ of an inch—which corresponds to about 0.9 foot difference of elevation. In the field there should be read, at each point whose elevation is desired, the aneroid, the time, and the temperature. These readings, corrected by the mean value of the correction between the aneroid and the mercurial, should then be combined with the reading of the mercurial (interpolated if necessary) for the times of the aneroid observations and the difference of elevation obtained. The field notes for the aneroid should be taken as shown in the first four columns of the tabular form. The "corrected aneroid" readings of column 5 are found by correcting the readings of column 3 by the mean difference between the mercurial and aneroid when compared at morning and night. Column 6 is a copy of the "corrected readings" from the office notes, interpolated when necessary for the proper time. Column 7 is similarly obtained. Col. 8 is obtained from cols. 4 and 5, and col. 9 from cols. 6 and 7, with the aid of Table XII. The correction for temperature (col. 11), which is generally small unless the difference of elevation is large, is obtained with the aid of Table XIII. The elevations in Table XII are elevations above an assumed datum plane, where under the given atmospheric conditions the mercurial reading would be 30″. Of course the position of this assumed plane changes with varying atmospheric conditions and so the elevations are to be considered as *relative* and their difference taken. [See the author's "Problems in the Use and Adjustment of Engineering In-

Digitized by Google

(Left-hand page of Notes.)

Time.	Place.	Aneroid.	Therm.	Corr. Aner.	Corr. Merc.
7:00	Office	29.628	73°	.....	29.755
7:10	Δ0	29.662	72°	29.789	29.748
7:30	saddle-back	29.374	63°	29.501	29.733
7:50	river cross.	29.548	70°	29.675	29.720

struments," Prob. 22.] Important points should be observed more than once if possible. Such duplicate observations will be found to give surprisingly concordant results even when a general fluctuation of atmospheric pressure so modifies the tabulated readings that an agreement is not at first apparent. Variations of pressure produced by high winds, thunder-storms, etc., will generally vitiate possible accuracy by this method. By "headquarters" is meant any place whose elevation above any given datum is known and where the mercurial may be placed and observed while observations within a range of several miles are made with the aneroid. If necessary, the elevation of a new headquarters may be determined by the above method, but there should be if possible several independent observations whose accordance will give a fair idea of their accuracy.

The above method should be neither slighted nor used for more than it is worth. When properly used, the errors are compensating rather than cumulative. When used, for example, to determine that a pass B is 260 feet higher than a determined bridge crossing at A which is six miles distant, and that another pass C is 310 feet higher than A and is ten miles distant, the figures, even with all necessary allowances for inaccuracy, will give an engineer a good idea as to the choice of route especially as affected by ruling grade. There is no comparison between the time and labor involved in obtaining the above information by barometric and by spirit-leveling methods, and *for reconnoissance purposes* the added accuracy of the spirit-leveling method is hardly worth its cost.

8. Horizontal measurements, bearings, etc. When there is no map which may be depended on, or when only a skeleton map is obtainable, a rapid survey, sufficiently accurate for the purpose, may be made by using a pocket compass for bearings and a telemeter, odometer, or pedometer for distances. The telemeter [stadia] is more accurate, but it requires a definite clear

Digitized by Google

(Right-hand page of Notes.)

Temp. at headqu.	Approx. field read.	Approx. headq. read.	Diff.	Corr. for temp.	Diff. elev.
—	—	—	—	—	—
75°	192	230	− 38	−(+ 2)	− 40
76	457	244	+213	+(+10)	+223
77	297	256	+ 41	+(+ 2)	+ 43

sight from station to station, which may be difficult through a wooded country. The odometer, which records the revolutions of a wheel of known circumference, may be used even in rough and wooded country, and the results may be depended on to a small percentage. The pedometer (or pace-measurer) depends for its accuracy on the actual movement of the mechanism for each pace and on the uniformity of the pacing. Its results are necessarily rough and approximate, but it may be used to fill in some intermediate points in a large skeleton map. A hand-level is also useful in determining the relative elevation of various topographical features which may have some bearing on the proper location of the road.

9. Importance of a good reconnoissance. The foregoing instruments and methods should be considered only as aids in exercising an educated common sense, without which a proper location cannot be made. The reconnoissance survey should command the best talent and the greatest experience available. If the general route is properly chosen, a comparatively low order of engineering skill can fill in a location which will prove a paying railroad property; but if the general route is so chosen that the ruling grades are high and the business obtained is small and subject to competition, no amount of perfection in detailed alignment or roadbed construction can make the road a profitable investment.

PRELIMINARY SURVEYS.

10. Character of survey. A preliminary railroad survey is properly a topographical survey of a belt of country which has been selected during the reconnoissance and within which it is estimated that the located line will lie. The width of this belt will depend on the character of the country. When a railroad is to follow a river having very steep banks the choice of location is sometimes limited at places to a very few feet of width

Digitized by Google

and the belt to be surveyed may be correspondingly narrowed. In very flat country the desired width may be only limited by the ability to survey points with sufficient accuracy at a considerable distance from what may be called the "backbone line" of the survey.

11. **Cross-section method.** This is the only feasible method in a wooded country, and is employed by many for all kinds of country. The *backbone* line is surveyed either by observing magnetic bearings with a compass or by carrying forward

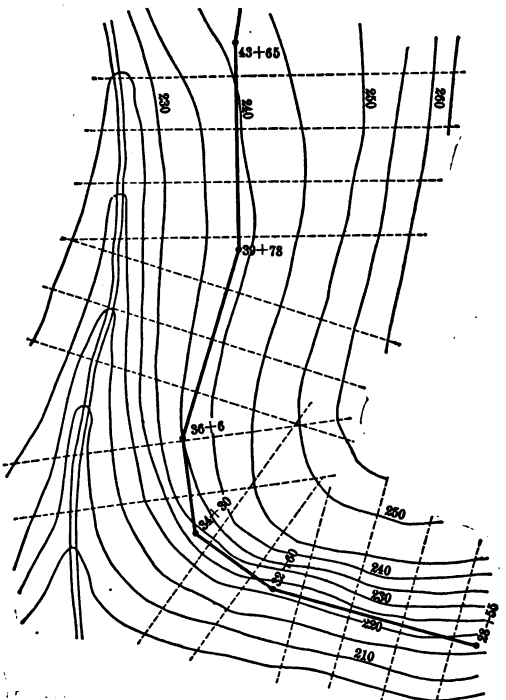


FIG. 4.

absolute azimuths with a transit. The compass method has the disadvantages of limited accuracy and the possibility of considerable local error owing to local attraction. On the other hand there are the advantages of greater simplicity, no necessity

Digitized by Google

for a back rodman, and the fact that the errors are purely local and not cumulative, and may be so limited, with care, that they will cause no vital error in the subsequent location survey. The transit method is essentially more accurate, but is liable to be more laborious and troublesome. If a large tree is encountered, either it must be cut down or a troublesome operation of offsetting must be used. If the compass is employed under these circumstances, it need only be set up on the far side of the tree and the former bearing produced. An error in reading a transit azimuth will be carried on throughout the survey. An error of only five minutes of arc will cause an offset of nearly eight feet in a mile. Large azimuth errors may, however, be avoided by immediately checking each new azimuth with a needle reading. It is advisable to obtain true azimuth at the beginning of the survey by an observation on the sun* or Polaris, and to check the azimuths every few miles by azimuth observations. Distances along the backbone line should be measured with a chain or steel tape and stakes set every 100 feet. When a course ends at a substation, as is usually the case, the remaining portion of the 100 feet should be measured along the next course. The level party should immediately obtain the elevations (to the nearest tenth of a foot) of all stations, and also of the lowest points of all streams crossed and even of dry gullies which would require culverts.

12. **Cross-sectioning.** It is usually desirable to obtain contours at five-foot intervals This may readily be done by the use of a Locke level (which should be held on top of a simple five-foot stick), a tape, and a rod ten feet in length graduated to feet and tenths. The method of use may perhaps be best explained by an example. Let Fig. 5 represent a section *perpendicular* to the survey line—such a section as would be made by the dotted lines in Fig. 4. *C* represents the station point. Its elevation as determined by the level is, say, 158.3 above datum. When the Locke level on its five-foot rod is placed at *C*, the level has an elevation of 163.3. Therefore when a point is found (as at *a*) where the level will read 3.3 on the rod, that point has an elevation of 160.0 and its distance from the center gives the position of the 160-foot contour. Leaving the long rod at that point (*a*), carry the level to some point (*b*) such that the level will sight at the *top* of the rod. *b* is then on the 165-

* For detailed methods of such determinations, see the author's "Problems in the Use and Adjustment of Engineering Instruments." Problems 35 and 36.

Digitized by Google

foot contour, and the *horizontal* distance *ab* added to the horizontal distance *ac* gives the position of that contour from the center. The contours on the lower side are found similarly. The first rod reading will be 8.3, giving the 155-foot contour.

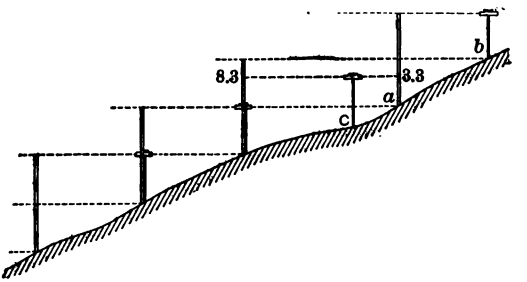


FIG. 5.

Plot the results in a note-book which is ruled in quarter-inch squares, using a scale of 100 feet per inch in both directions. Plot the work UP the page; then when looking ahead along the line, the work is properly oriented. When a contour crosses

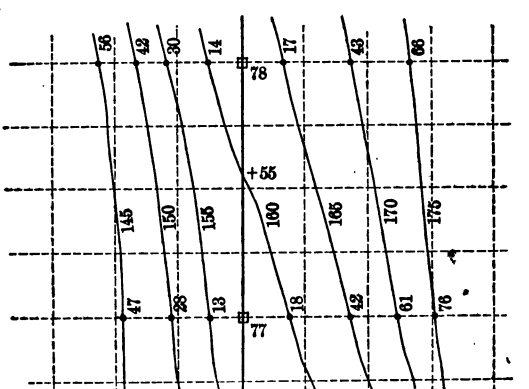


FIG. 6.

the survey line, the place of crossing may be similarly determined. If the ground flattens out so that five-foot contours are very far apart, the absolute elevations of points at even fifty-

Digitized by Google

foot distances from the center should be determined. The method is exceedingly rapid. Whatever error or inaccuracy occurs is confined in its effect to the one station where it occurs. The work being thus plotted in the field, unusually irregular topography may be plotted with greater certainty and no great error can occur without detection. It would even be possible by this method to detect a gross error that might have been made by the level party

13. Stadia method. This method is best adapted to fairly open country where a "shot" to any desired point may be taken without clearing. The *backbone* survey line is the same as in the previous method except that each course is limited to the practicable length of a stadia sight. The distance between stations should be checked by foresight and backsight—also the vertical angle. Azimuths should be checked by the needle. Considering the vital importance of leveling on a railroad survey it might be considered desirable to run a line of levels over the stadia stations in order that the leveling may be as precise as possible; but when it is considered that a preliminary survey is a somewhat hasty survey of a route that *may* be abandoned, and that the errors of leveling by the stadia method (which are compensating) may be so minimized that no proposed route would be abandoned on account of such small error, and that the effect of such an error may be easily neutralized by a slight change in the location, it may be seen that excessive care in the leveling of the preliminary survey is hardly justifiable.

Since the students taking this work are assumed to be familiar with the methods of stadia topographical surveys, this part of the subject will not be further elaborated.

14. "First" and "Second" preliminary surveys. Some engineers advocate two preliminary surveys. When this is done, the first is a very rapid survey, made perhaps with a compass, and is only a better grade of reconnoissance. Its aim is to rapidly develop the facts which will decide for or against any proposed route, so that if a route is found to be unfavorable another more or less modified route may be adopted without having wasted considerable time in the survey of useless details. By this time the student should have grasped the fundamental idea that both the reconnoissance and preliminary surveys are not surveys of *lines* but of *areas;* that their aim is to survey only those topographical features which would have a deter-

Digitized by Google

mining influence on any railroad line which might be constructed through that particular territory, and that the value of a locating engineer is largely measured by his ability to recognize those determining influences with the least amount of work from his surveying corps. Frequently too little time is spent on the comparative study of preliminary lines. A line will be hastily decided on after very little study; it will then be surveyed with minute detail and estimates carefully worked up, and the claims of any other suggested route will then be handicapped, if not disregarded, owing to an unwillingness to discredit and throw away a large amount of detailed surveying. The cost of two or three extra preliminary surveys (*at critical sections* and not over the whole line) is utterly insignificant compared with the probable improvement in the "operating value" of a line located after such a comparative study of preliminary lines.

LOCATION SURVEYS.

15. "Paper location." When the preliminary survey has been plotted to a scale of 200 feet per inch and the contours drawn in, a study may be made for the location survey. Disregarding for the present the effect on location of transition curves, the alignment may be said to consist of straight lines (or "tangents") and circular curves. The "paper location" therefore consists in plotting on the preliminary map a succession of straight lines which are tangent to the circular curves connecting them. The determining points should first be considered. Such points are the termini of the road, the lowest practicable point over a summit, a river-crossing, etc. So far as is possible, having due regard to other considerations, the road should be a "surface" road, i.e., the cut and fill should be made as small as possible. The maximum permissible grade must also have been determined and duly considered. The method of location differs radically according as the lines joining the determining points have a very low grade or have a grade that approaches the maximum permissible. With very low natural grades it is only necessary to strike a proper balance between the requirements for easy alignment and the avoidance of excessive earthwork. When the grade between two determined points approaches the maximum, a study of the location may be begun by finding a strictly surface line which will connect those

Digitized by Google

points with a line at the given grade. For example, suppose the required grade is 1.6% and that the contours are drawn at 5-foot intervals. It will require 312 feet of 1.6% grade to rise 5 feet. Set a pair of dividers at 312 feet and step off this interval on successive contours. This line will in general be very irregular, but in an easy country it may lie fairly close to the proper location line, and even in difficult country such a surface line will assist greatly in selecting a suitable location. When the larger part of the line will evidently consist of tangents, the tangents should be first located and should then be connected by suitable curves. When the curves predominate, as they generally will in mountainous country, and particularly when the line is purposely lengthened in order to reduce the grade, the curves should be plotted first and the tangents may then be drawn connecting them. Considering the ease with which such lines may be drawn on the preliminary map, it is frequently advisable, after making such a paper location, to begin all over, draw a new line over some specially difficult section and compare results. Profiles of such lines may be readily drawn by noting their intersection with each contour crossed. Drawing on each profile the required grade line will furnish an approximate idea of the *comparative* amount of earthwork required. After deciding on the paper location, the length of each tangent, the central angle (see § 21), and the radius of each curve should be measured as accurately as possible. Since a slight error made in such measurements, taken from a map with a scale of 200 feet per inch, would by accumulation cause serious discrepancies between the plotted location and the location as afterward surveyed in the field, frequent tie lines and angles should be determined between the plotted location line and the preliminary line, and the location should be altered, as may prove necessary, by changing the length of a tangent or changing the central angle or radius of a curve, so that the agreement of the check-points will be sufficiently close. The errors of an inaccurate preliminary survey may thus be easily neutralized (see § 33). When the preliminary line has been properly run, its "backbone" line will lie very near the location line and will probably cross it at frequent intervals, thus rendering it easy to obtain short and numerous tie lines.

16. Surveying methods. A transit should be used for alignment, and only precise work is allowable. The transit stations

Digitized by Google

should be centered with tacks and should be tied to witness-stakes, which should be located outside of the range of the earth-work, so that they will neither be dug up nor covered up. All original property lines lying within the limits of the right of way should be surveyed with reference to the location line, so that the right-of-way agent may have a proper basis for settlement. When the property lines do not extend far outside of the required right of way they are frequently surveyed completely.

The leveler usually reads the target to the nearest thousandth of a foot on turning-points and bench-marks, but reads to the nearest tenth of a foot for the elevation of the ground at stations. Considering that $\frac{1}{1000}$ of a foot has an angular value of about

FORM OF NOTES.

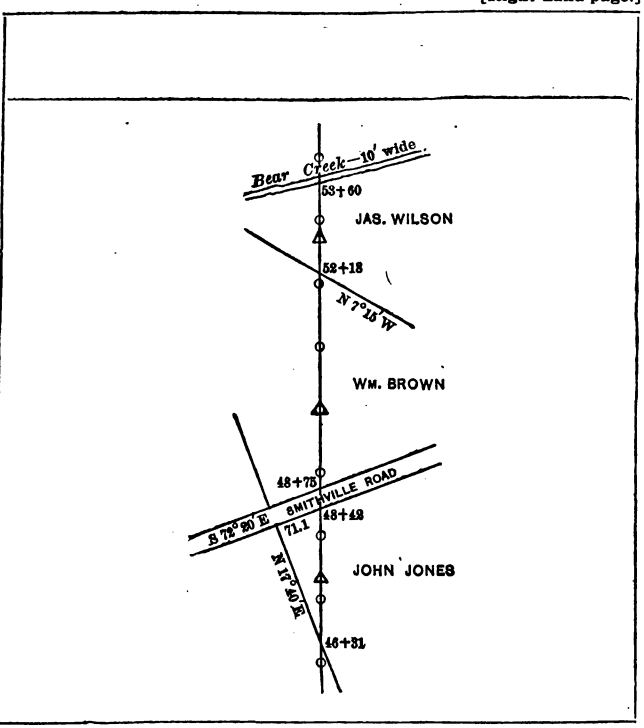
[Left-hand page.]

Sta.	Alignment.	Vernier.	Tangential Deflection.	Calculated Bearing.	Needle.
54					
53 ⊙ +72.2	P.T.	9° 11′	18° 22′	N 54° 48′ E	N 62° 15′ E
52		7 57			
51	[3° 24′ curve to right for 18° 22′; tang. dist., 272.5]	6 15			
⊙ 50		4 33			
49		2 51			
48		1 09			
⊙ +32 47	P.C.	0°			
46				N 36° 26′ E	N 44° 0′ E

Digitized by Google

one second at a distance of 200 feet, and that one division of a level-bubble is usually about 30 seconds, it may be seen that it is a useless refinement to read to thousandths unless corresponding care is taken in the use of the level. The leveler should also locate his bench-marks outside of the range of earthwork. A knob of rock protruding from the ground affords an excellent mark. A large nail, driven in the roots of a tree, which is not to be disturbed, is also a good mark. These marks should be clearly described in the note-book. The leveler should obtain the elevation of the ground at all station-points; also at all sudden breaks in the profile line, determining also the distance of these breaks from the previous even station. This will in-

[Right-hand page.]



Digitized by Google

clude the position and elevation of all streams, and even dry gullies, which are crossed

Measurements should preferably be made with a steel tape, care being taken on steep ground to insure horizontal measurements. Stakes are set each 100 feet, and also at the beginning and end of all curves. Transit-points (sometimes called "plugs" or "hubs") should be driven flush with the ground, and a "witness-stake," having the "number" of the station, should be set three feet to the right. For example, the witness-stake might have on one side "137+69.92," and on the other side "P C 4° R," which would signify that the transit hub is 69.92 feet beyond station 137, or 13769.92 feet from the beginning of the line, and also that it is the "point of curve" of a "4° curve" which turns to the *right.*

Alignment. The alignment is evidently a part of the location survey, but, on account of the magnitude and importance of the subject, it will be treated in a separate chapter.

17. **Form of Notes.** Although the Form of Notes cannot be thoroughly understood until after curves are studied, it is here introduced as being the most convenient place. The right-hand page should have a sketch showing all roads, streams, and property lines crossed with the bearings of those lines. This should be drawn to a scale of 100 feet per inch—the quarter-inch squares which are usually ruled in note-books giving convenient 25-foot spaces. This sketch will always be more or less distorted on curves, since the center line is always shown as *straight* regardless of curves. The station points ("Sta." in first column, left-hand page) should be placed opposite to their sketched positions, which means that even stations will be recorded on every *fourth* line. This allows three intermediate lines for substations, which is ordinarily more than sufficient. The notes should read UP the page, so that the sketch will be properly oriented when looking ahead along the line The other columns on the left-hand page will be self-explanatory when the subject of curves is understood. If the "calculated bearings" are based on azimuthal observations, their agreement (or *constant* difference) with the needle readings will form a valuable check on the curve calculations and the instrumental work.

Digitized by Google

CHAPTER II.

ALIGNMENT.

In this chapter the alignment of the *center line* only of a pair of rails is considered. When a railroad is crossing a summit in the grade line, although the horizontal projection of the alignment may be straight, the vertical projection will consist of two sloping lines joined by a curve. When a curve is on a grade, the center line is really a spiral, a curve of double curvature, although its horizontal projection is a circle. The center line therefore consists of straight lines and curves of single and double curvature. The simplest method of treating them is to consider their horizontal and vertical projections separately. In treating simple, compound, and transition curves, only the horizontal projections of those curves will be considered.

SIMPLE CURVES.

18. Designation of curves. A curve may be designated either by its radius or by the angle subtended by a chord of unit length. Such an angle is known as the "degree of curve" and is indicated by D. Since the curves that are practically used have very long radii, it is generally impracticable to make any use of the actual center, and the curve is located without reference to it. If AB in Fig. 4 represents a unit chord (C) of a curve of radius R, then by the above definition the angle AOB equals D. Then

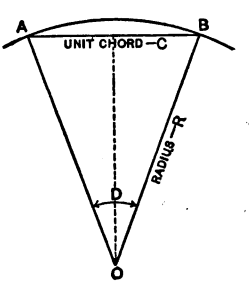


FIG. 7.

$$AO \sin \tfrac{1}{2}D = \tfrac{1}{2}AB = \tfrac{1}{2}C.$$

$$\therefore R = \frac{\frac{1}{2}C}{\sin \frac{1}{2}D}, \quad \ldots\ldots (1)$$

19-33

Digitized by Google

or, by inversion,

$$\sin \tfrac{1}{2}D = \frac{C}{2R}. \quad \ldots\ldots\ldots \quad (2)$$

The unit chord is variously taken throughout the world as 100 feet, 66 feet, and 20 meters. In the United States 100 feet is invariably used as the unit chord length, and throughout this work it will be so considered. Table I has been computed on this basis. It gives the radius, with its logarithm, of all curves from a 0° 01′ curve up to a 10° curve, varying by single minutes. The sharper curves, which are seldom used, are given with larger intervals.

An approximate value of R may be readily found from the following simple rule, which should be memorized:

$$R = \frac{5730}{D}.$$

Although such values are not mathematically correct, since R does not strictly vary inversely as D, yet the resulting value is within a tenth of one per cent for all commonly used values of R, and is sufficiently close for many purposes, as will be shown later.

19. Length of a subchord. Since it is impracticable to measure along a curved arc, curves are always measured by laying off 100-foot chord lengths. This means that the actual arc is always a little longer than the chord. It also means that a *subchord* (a chord shorter than the unit length) will be a little longer than the ratio of the angles subtended would call for. The truth of this may be seen without calculation by noting that two equal subchords, each subtending the angle $\frac{1}{2}D$, will evidently be slightly longer than 50 feet each. If c be the length of a subchord subtending the angle d, then, as in Eq. 2,

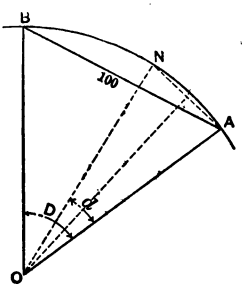


FIG. 8.

$$\sin \tfrac{1}{2}d = \frac{c}{2R},$$

Digitized by Google

or, by inversion,

$$c = 2R \sin \tfrac{1}{2}d. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The *nominal length* of a subchord $= 100\frac{d}{D}$ For example, a *nominal* subchord of 40 feet will subtend an angle of $\frac{40}{100}$ of $D°$; its *true length* will be slightly more than 40 feet, and may be computed by Eq. 3. The *difference* between the nominal and true lengths is maximum when the subchord is about 57 feet long, but with the low degrees of curvature ordinarily used the difference may be neglected. With a 10° curve and a nominal chord length of 60 feet, the true length is 60.049 feet. Very sharp curves should be laid off with 50-foot or even 25-foot chords (nominal length). In such cases especially the true lengths of these subchords should be computed and used instead of the nominal lengths.

20. **Length of a curve.** The length of a curve is always indicated by the quotient of $100\Delta \div D$. If the quotient of $\Delta \div D$ is a whole number, the length as thus indicated is the true length—*measured in 100-foot chord lengths.* If it is an odd number or if the curve begins and ends with a subchord (even though $\Delta \div D$ is a whole number), theoretical accuracy requires that the *true* subchord lengths shall be used, although the difference may prove insignificant. The length of the arc (or the mean length of the two rails) is therefore always in excess of the length as given above. Ordinarily the amount of this excess is of no practical importance. It simply adds an insignificant amount to the length of rail required.

Example. Required the nominal and true lengths of a 3° 45′ curve having a central angle of 17° 25′. First reduce the degrees and minutes to decimals of a degree. $(100 \times 17° 25') \div 3° 45' = 1741.667 \div 3.75 = 464.444$. The curve has four 100-foot chords and a nominal chord of 64.444 The true chord should be 64.451. The actual arc is

$$17°.4167 \times \frac{\pi}{180°} \times R = 464.527$$

The excess is therefore $464.527 - 464.451 = 0.076$ foot.

21. **Elements of a curve.** Considering the line as running from A toward B, the beginning of the curve, at A, is called the *point of curve* (*PC*). The other end of the *curve*, at B, is

Digitized by Google

called the *point of tangency* (*PT*). The intersection of the tangents is called the *vertex* (*V*). The angle made by the tangents at *V*, which equals the angle made by the radii to the extremities of the curve, is called the *central angle* (Δ). *AV* and *BV*, the two equal tangents from the vertex to the *PC* and *PT*, are called the *tangent distances* (*T*). The chord *AB* is called the *long chord* (*LC*). The intercept *HG* from the middle of the long chord to the middle of the arc is called the *middle ordinate* (*M*). That part of the secant *GV* from the middle of the arc to the vertex is called the *external distance* (*E*). From the figure it is very easy to derive the following frequently used relations:

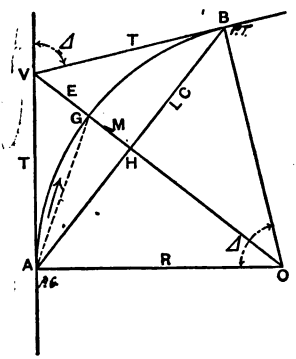


FIG. 9.

$$T = R \tan \tfrac{1}{2}\Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$LC = 2R \sin \tfrac{1}{2}\Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$M = R \operatorname{vers} \tfrac{1}{2}\Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$E = R \operatorname{exsec} \tfrac{1}{2}\Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

22. Relation between *T*, *E*, and Δ. Join *A* and *G* in Fig. 9. The angle $VAG = \frac{1}{4}\Delta$, since it is measured by one half of the arc *AG* between the secant and tangent. $AGO = 90° - \frac{1}{4}\Delta$.

$$AV : VG :: \sin AGV : \sin VAG;$$

$$\sin AGV = \sin AGO = \cos \tfrac{1}{4}\Delta;$$

$$T : E :: \cos \tfrac{1}{4}\Delta : \sin \tfrac{1}{4}\Delta;$$

$$T = E \cot \tfrac{1}{4}\Delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The same relation may be obtained by dividing Eq. 4 by Eq. 7, since $\tan \alpha \div \operatorname{exsec} \alpha = \cot \frac{1}{2}\alpha$.

23. Elements of a 1° curve. From Eqs. 1 to 8 it is seen that the elements of a curve vary directly as *R*. It is also seen to be very nearly true that *R* varies inversely as *D*. If the elements of a 1° curve for various central angles are calculated and tabulated, the elements of a curve of *D*° curvature may approximately found by dividing by *D* the corresponding elements of a 1° curve having the same central angle. For

Digitized by Google

central angles and low degrees of curvature the errors involved by the approximation are insignificant, and even for larger angles the errors are so small that *for many purposes* they may be disregarded.

In Table II is given the value of the tangent distances, external distances, and long chords for a 1° curve for various central angles The student should familiarize himself with the degree of approximation involved by solving a large number of cases under various conditions by the exact and by the approximate methods, in order that he may know when the approximate method is sufficiently exact for the intended purpose. The approximate method also gives a ready check on the exact method.

24. Exercises. (*a*) What is the tangent distance of a 4° 20′ curve having a central angle of 18° 24′?

(*b*) Given a 3° 30′ curve and a central angle of 16° 20′, how far will the curve pass from the vertex? [Use Eq. 7.]

(*c*) An 18° curve is to be laid off using 25-foot (nominal) chord lengths. What is the true length of the subchords?

(*d*) Given two tangents making a central angle of 15° 24′. It is desired to connect these tangents by a curve which shall pass 16.2 feet from their intersection. How far down the tangent will the curve begin and what will be its radius? (Use Eq. 8 and then use Eq. 4 inverted.)

25. Curve location by deflections. The angle between a secant and a tangent (or between two secants intersecting on an arc) is measured by one half of the intercepted arc. Beginning at the *PC* (*A* in Fig. 10), if the first chord is to be a full chord we may deflect an angle VAa ($=\frac{1}{2}D$), and the point *a*, which is 100 feet from *A*, is a point on the curve. For the next station, *b*, deflect an *additional* angle bAa ($=\frac{1}{2}D$) and, with one end of the tape at *a*, swing the other end until the 100-foot point is on the line *Ab*. The point *b* is then on the curve. If the final chord *cB* is a subchord, its *additional deflection* ($\frac{1}{2}d$) is something less than $\frac{1}{2}D$.

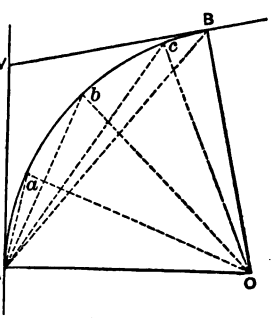


FIG. 10.

The last *deflection* (BAV) :-

Digitized by Google

of course $\frac{1}{2}\Delta$. It is particularly important, when a curve begins or ends with a subchord and the deflections are odd quantities, that the last additional deflection should be carefully computed and added to the previous deflection, to check the mathematical work by the agreement of this last computed deflection with $\frac{1}{2}\Delta$.

Example. Given a 3° 24′ curve having a central angle of 18° 22′ and beginning at sta. 47+32, to compute the deflections. The nominal length of curve is 18° 22′ ÷ 3° 24′ = 18.367 ÷ 3.40 = 5.402 stations or 540.2 feet. The curve therefore ends at sta. 52+72.2. The deflection for sta. 48 is $\frac{68}{100} \times \frac{1}{2}(3° 24′)$ = 0.68 × 1°.7 = 1°.156 = 1° 09′ nearly. For each additional 100 feet it is 1° 42′ additional. The final additional deflection for the final subchord of 72.2 feet is

$$\frac{72.2}{100} \times \tfrac{1}{2}(3° 24′) = 1°.2274 = 1° 14′ \text{ nearly.}$$

The deflections are

P. C ... Sta.	47+32	.....	0°
	48	..... 0°	+ 1° 09′ = 1° 09′
	49	..... 1° 09′	+ 1° 42′ = 2° 51′
	50	..... 2° 51′	+ 1° 42′ = 4° 33′
	51	..... 4° 33′	+ 1° 42′ = 6° 15′
	52	..... 6° 15′	+ 1° 42′ = 7° 57′
P. T.	52+72.2.	..... 7° 57′	+ 1° 14′ = 9° 11′

As a check 9° 11′ = ½(18° 22′) = ½Δ. (See the Form of Notes in § 17.)

26. Instrumental work. It is generally impracticable to locate more than 500 to 600 feet of a curve from one station. Obstructions will sometimes require that the transit be moved up every 200 or 300 feet. There are two methods of setting off the angles when the transit has been moved up from the *PC*.

(a) The transit may be sighted at the previous transit station with a reading on the plates equal to the deflection angle from that station to the station occupied, but with the angle set off on the *other* side of 0°, so that when the telescope is turned to 0° it will sight along the tangent at the station occupied. Plunging the telescope, the forward stations may be set off by deflecting the proper deflections from the tangent at the station occupied

Digitized by Google

This is a very common method and, when the degree of curvature is an even number of degrees and when the transit is only set at even stations, there is but little objection to it. But the degree of curvature is sometimes an odd quantity, and the exigencies of difficult location frequently require that substations be occupied as transit stations. Method (*a*) will then require the recalculation of all deflections for each new station occupied. The mathematical work is largely increased and the probability of error is very greatly increased and not so easily detected. Method (*b*) is just as simple as method (*a*) even for the most simple cases, and for the more difficult cases just referred to the superiority is very great.

(b) Calculate the deflection for each station and substation throughout the curve as though the whole curve were to be located from the *PC*. The computations may thus be completed and *checked* (as above) before beginning the instrumental work. If it unexpectedly becomes necessary to introduce a substation at any point, its deflection from the *PC* may be readily interpolated. The stations actually set from the *PC* are located as usual.

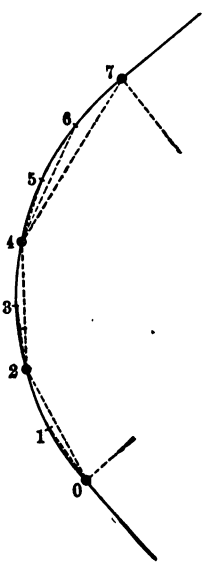


FIG. 11.

RULE. **When the transit is set on any forward station, backsight to ANY previous station with the plates set at the deflection angle for the station sighted at. Plunge the telescope and sight at any forward station with the deflection angle originally computed for that station.** When the plates read the deflection angle for the station occupied, the telescope is sighting along the tangent at that station—which is the method of getting the forward tangent when occupying the *PT*. Even though the station occupied is an unexpected substation, when the instrument is properly oriented at that station, the angle reading for *any* station, forward or back, is that originally computed for it from the *PC*. In difficult work, where there are obstructions, a valuable check on the accuracy may be found by sighting backward at *any* visible station and noting whether

Digitized by Google

its deflection agrees with that originally computed. As a numerical illustration, assume a 4° curve, with 28° curvature, with stations 0, 2, 4, and 7 occupied. After setting stations 1 and 2, set up the transit at sta. 2 and backsight to sta. 0 with the deflection for sta. 0, which is 0°. The reading on sta. 1 is 2°; when the reading is 4° the telescope is tangent to the curve, and when sighting at 3 and 4 the deflections will be 6° and 8°. Occupy 4; sight to 2 with a reading of 4°. When the reading is 8° the telescope is tangent to the curve and, by plunging the telescope, 5, 6, and 7 may be located with the originally computed deflections of 10°, 12°, and 14°. When occupying 7 a backsight may be taken to any visible station with the plates reading the deflection for that station; then when

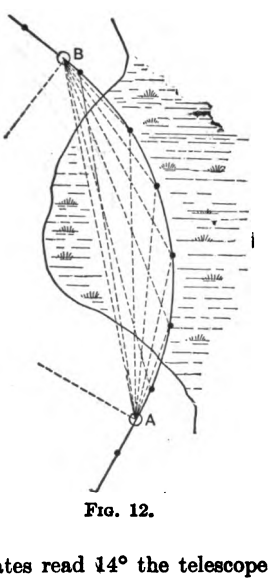


Fig. 12.

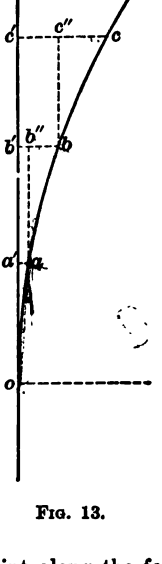


Fig. 13.

the plates read 14° the telescope will point along the forward tangent.

The location of curves by deflection angles is the normal method. A few other methods, to be described, should be considered as exceptional.

Digitized by Google

27. Curve location by two transits. A curve might be located more or less on a swamp where accurate chaining would be exceedingly difficult if not impossible. The long chord AB (Fig. 12) may be determined by triangulation or otherwise, and the elements of the curve computed, including (possibly) subchords at each end. The deflection from A and B to each point may be computed. A rodman may then be sent (by whatever means) to locate long stakes at points determined by the simultaneous sightings of the two transits.

28. Curve location by tangential offsets. When a curve is very flat and no transit is at hand the following method may be used (see Fig. 13): Produce the back tangent as far forward as necessary. Compute the ordinates Oa', Ob', Oc', etc., and the abscissæ $a'a$, $b'b$, $c'c$, etc. If Oa is a full station (100 feet), then

$$\left.\begin{aligned} Oa' = Oa' &= 100 \cos \tfrac{1}{2}D, \text{ also} = R \sin D; \\ Ob' = Oa' + a'b' &= 100 \cos \tfrac{1}{2}D + 100 \cos \tfrac{3}{2}D, \\ &\quad \text{also} = R \sin 2D; \\ Oc' = Oa' + a'b' + b'c' &= 100(\cos \tfrac{1}{2}D + \cos \tfrac{3}{2}D + \cos \tfrac{5}{2}D), \\ &\quad \text{also} = R \sin 3D; \end{aligned}\right\} \quad (9)$$

etc.

$$\left.\begin{aligned} a'a = &\quad 100 \sin \tfrac{1}{2}D, \text{ also} = R \operatorname{vers} D; \\ b'b = a'a + b''b &= 100 \sin \tfrac{1}{2}D + 100 \sin \tfrac{3}{2}D, \\ &\quad \text{also} = R \operatorname{vers} 2D; \\ c'c = b'b + c''c &= 100(\sin \tfrac{1}{2}D + \sin \tfrac{3}{2}D + \sin \tfrac{5}{2}D), \\ &\quad \text{also} = R \operatorname{vers} 3D; \end{aligned}\right\} \quad (10)$$

etc.

The functions $\frac{1}{2}D$, $\frac{3}{2}D$, etc., may be more conveniently used *without* logarithms, by adding the several *natural* trigonometrical functions and pointing off two decimal places. It may also be noted that Ob' (for example) is one half of the long chord for four stations; also that $b'b$ is the middle ordinate for four stations. If the engineer is provided with tables giving the long chords and middle ordinates for various degrees of curvature, these quantities may be taken (perhaps by interpolation) from such tables.

If the curve begins or ends at a substation, the angles and The s will be correspondingly altered. The modifications may

Digitized by Google

be readily deduced on the same principles as above, and should be worked out as an exercise by the student.

In Table II are given the long chords for a 1° curve for various values of Δ. Dividing the value as given by the degree of the curve, we have an approximate value which is amply close for low degrees of curvature, especially for laying out curves without a transit. For example, given a 4° 30′ curve, required the ordinate Oc'. This is evidently one half of a chord of six stations, with $\Delta = 27°$. Dividing 2675.1 (which is the long chord of a 1° curve with $\Delta = 27°$) by 4.5 we have 594.47; one half of this is the required ordinate, $Oc' = 297.23$. The exact value is 297.31, an excess of .08, or less than .03 of 1%. The true values are always slightly in excess of the value as computed from Table II.

Exercise. A 3° 40′ curve begins at sta. 18+70 and runs to sta. 23+60. Required the tangential offsets and their corresponding ordinates. The first ordinate $= 30 \cos \frac{1}{2}(\frac{3.0}{1.0.0} \times 3° 40') = 30 \times .99995 = 29.9985$; the offset $= 30 \sin 0° 33' = 30 \times .0096 = 0.288$. For the second full station (sta. 20) the ordinate = ½ long chord for $\Delta = 2(1° 06' + 3° 40')$ with $D = 3° 40'$. Dividing 476.12, from Table II, by $3\frac{2}{3}$, we have 129.85. Otherwise, by Eq. 9, the ordinate $= 30 \times \cos 0° 33' + 100 \cos (1° 06' + 1° 50') = 30.00 + 99.87 = 129.87$. The offset for sta. $20 = 30 \sin 0° 33' + 100 \sin (1° 06' + 1° 50') = 0.288 + 5.12 = 5.41$. Work out similarly the ordinates and offsets for sta. 21, 22, 23, and 23+60.

29. **Curve location by middle ordinates.** Take first the simpler case when the curve begins at an even station. If we consider (in Fig. 14) the curve produced back to z, the chord $za = 2 \times 100 \cos \frac{1}{2}D$, $A'a = 100 \cos \frac{1}{2}D$, and $A'A = am = zn = 100 \sin \frac{1}{2}D$. Set off AA' perpendicular to the tangent and $A'a$ parallel to the tangent. $AA' = aa' = bb' = cc'$, etc. $= 100 \sin \frac{1}{2}D$. Set off aa' perpendicular to $a'A$. Produce Aa' until $a'b = A'a$, thus determining b. Succeeding points of the curve may thus be determined indefinitely.

Suppose the curve begins with a subchord. As before $ra = Am' = c' \cos \frac{1}{2}d'$, and $rA = am' = c' \sin \frac{1}{2}d'$. Also $sz = An' = c'' \cos \frac{1}{2}d''$, and $sA = zn' = c'' \sin \frac{1}{2}d''$, in which $(d' + d'') = D$. The points z and a being determined on the ground, aa' may be computed and set off as before and the curve continued in-

Digitized by Google

full stations. A subchord at the end of the curve may be located by a similar process.

30. Curve location by offsets from the long chord. (Fig. 16.) Consider at once the general case in which the curve commences with a subchord (curvature, d'), continues with one or more full

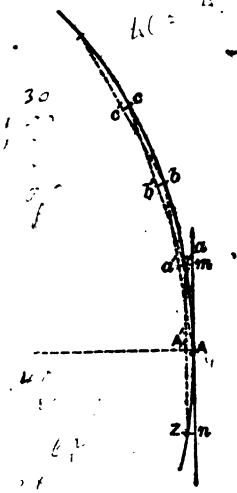


FIG. 14.

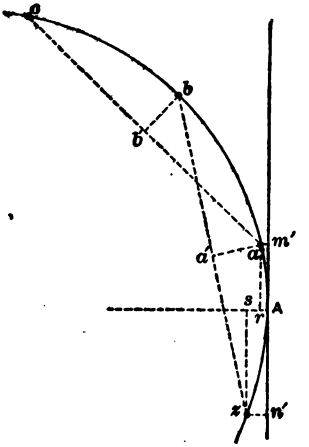


FIG. 15.

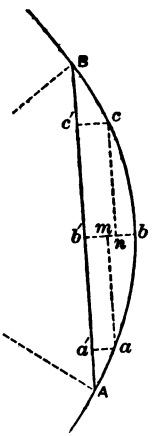


FIG. 16.

chords (curvature of each, D), and ends with a subchord with curvature d''. The numerical work consists in computing first AB, then the various abscissæ and ordinates. $AB = 2R \sin \frac{1}{2}\Delta$.

$$\left.\begin{aligned}
Aa' &= Aa' && = e' \cos \tfrac{1}{2}(\Delta - d');\\
Ab' &= Aa' + a'b' && = c' \cos \tfrac{1}{2}(\Delta - d') + 100 \cos \tfrac{1}{2}(\Delta - 2d' - D);\\
Ac' &= Aa' + a'b' + b'c' && = c' \cos \tfrac{1}{2}(\Delta - d') + 100 \cos \tfrac{1}{2}(\Delta - 2d' - D)\\
& && \quad + 100 \cos \tfrac{1}{2}(\Delta - 2d'' - D);\\
\text{also}\\
&= AB - Bd' && = 2R \sin \tfrac{1}{2}\Delta - c'' \cos \tfrac{1}{2}(\Delta - d'').
\end{aligned}\right\} \quad (11)$$

$$\left.\begin{aligned}
a'a &= a'a && = e' \sin \tfrac{1}{2}(\Delta - d');\\
b'b &= a'a + mb = c' && \sin \tfrac{1}{2}(\Delta - d') + 100 \sin \tfrac{1}{2}(\Delta - 2d' - D);\\
c'c &= b'b - nb = c' && \sin \tfrac{1}{2}(\Delta - d') + 100 \sin \tfrac{1}{2}(\Delta - 2d' - D)\\
& && \quad - 100 \sin \tfrac{1}{2}(\Delta - 2d'' - D);\\
\text{also} & && = c'' \sin \tfrac{1}{2}(\Delta - d'').
\end{aligned}\right\} \quad (12)$$

The above formulæ are considerably simplified when the

Digitized by Google

curve begins and ends at even stations. When the curve is very long a regular law becomes very apparent in the formation of all terms between the first and last. There are too few terms in the above equations to show the law.

31. Use and value of the above methods. The chief value of the above methods lies in the possibility of doing the work without a transit. The same principles are sometimes employed, even when a transit is used, when obstacles prevent the use of the normal method (see § 32, *c*). If the terminal tangents have already been accurately determined, these methods are useful to locate points of the curve when rigid accuracy is not essential. Track foremen frequently use such methods to lay out unimportant sidings, especially when the engineer and his transit are not at hand. Location by tangential offsets (or by offsets from the long chord) is to be preferred when the curve is flat (i.e., has a small central angle Δ) and there is no obstruction along the tangent, or long chord. Location by middle ordinates may be employed regardless of the length of the curve, and in cases when both the tangents and the long chord are obstructed. The above methods are but samples of a large number of similar methods which have been devised. The choice of the particular method to be adopted must be determined by the local conditions.

32. Obstacles to location. In this section will be given only a few of the principles involved in this class of problems, with illustrations. The engineer must decide, in each case, which is the best method to use. It is frequently advisable to devise a special solution for some particular case.

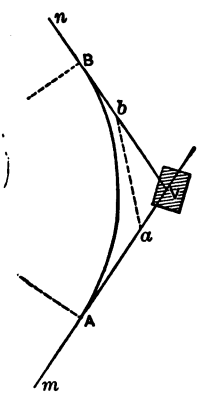


FIG. 17.

a. When the vertex is inaccessible. As shown in § 26, it is not absolutely essential that the vertex of a curve should be located on the ground. But it is very evident that the angle between the terminal tangents is determined with far less probable error if it is measured by a single measurement at the vertex rather than as the result of numerous angle measurements along the curve, involving several positions of the transit and comparatively short sights. Some-

Digitized by Google

times the location of the tangents is already determined on the ground (as by bn and am, Fig. 17), and it is required to join the tangents by a curve of given radius. *Method.* Measure ab and the angles Vba and baV. Δ is the sum of these angles. The distances bV and aV are computable from the above data. Given Δ and R, the tangent distances are computable, and then Bb and aA are found by subtracting bV and aV from the tangent distances. The curve may then be run from A, and the work may be checked by noting whether the curve as run ends at B—previously located from b.

Example. Assume $ab = 546\,82$; angle $a = 15° 18'$; angle $b = 18° 22'$; $D = 3° 40'$; required aA and bB.

$\Delta = 15° 18' + 18° 22' = 33° 40'$

Eq. (4)

	R $(3° 40')$	$3.1939\bar{2}$
	$\tan \frac{1}{2}\Delta = \tan 16° 50'$	9.48080
	$T = 472.85$	$2.6747\bar{2}$

$aV = ab \dfrac{\sin 18° 22'}{\sin 33° 40'}$

ab	$2.7378\bar{4}$
$\log \sin 18° 22'$	$9.4984\bar{4}$
co-log $\sin 33° 40'$	0.25621
$aV = 310.81$	$2.4925\bar{0}$

$AV = 472.85$

$aA = 162.04$

$bV = ab \dfrac{\sin 15° 18'}{\sin 33° 40'}$

ab	$2.7378\bar{4}$
$\log \sin 15° 18'$	$9.4213\bar{9}$
co-log $\sin 33° 40'$	0.25621
$bV = 260.29$	$2.4154\bar{5}$

$BV = 472.85$

$bB = 212.56$

b. When the point of curve (or point of tangency) is inaccessible. At some distance (As, Fig. 18) an unobstructed line pn may be run parallel with AV. $nv = py = As = R$ vers a.

$\therefore$ vers $a = As \div R$.

$ns = ps = R \sin a$.

Digitized by Google

At y, which is at a distance ps back from the *computed* position of A, make an offset sA to p. Run pn parallel to the tangent. A tangent to the curve at n makes an angle of a with np. From n the curve is run in as usual

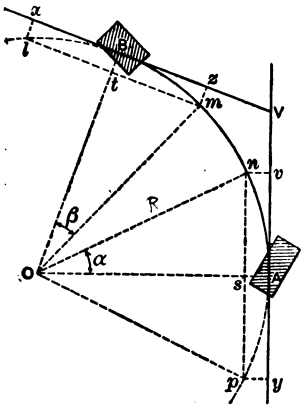


Fig. 18.

If the point of tangency is obstructed, a similar process, somewhat reversed, may be used. β is that portion of Δ still to be laid off when m is reached. $tm = tl = R \sin \beta$. $mz = tB = lx = R$ vers β.

c. When the central part of the curve is obstructed. a is the central angle between two points of the curve between which a chord may be run. a may equal *any* angle, but it is preferable that a should be a multiple of D, the degree of curve, and that the points m and n should be on even stations. $mn = 2R \sin \frac{1}{2}a$. A point s may be located by an offset ks from the chord mn by a similar method to that outlined in § 30.

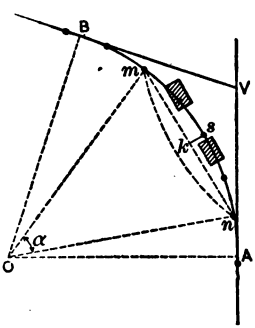


Fig. 19.

The device of introducing the dotted curve mn having the same radius of curvature as the other, although neither necessary nor advisable in the case shown in Fig. 19, is sometimes the best method of surveying around an obstacle. The offset from any point on the dotted curve to the corresponding point on the true curve is twice the "ordinate to the long chord," as computed in § 30.

33. Modifications of location. The following methods may be used in allowing for the discrepancies between the "paper location" based on a more or less rough preliminary survey and the more accurate instrumental location. (See § 15.) They are

Digitized by Google

also frequently used in locating new parallel tracks and modifying old tracks.

a. To move the forward tangent parallel to itself a distance x, the point of curve (A) remaining fixed. (Fig. 20.)

$$V'h = B'r = x'.$$

$$VV' = \frac{V'h}{\sin hVV'} = \frac{x}{\sin A}. \quad \ldots\ldots \quad (13)$$

$$AV' = AV + VV'.$$

The triangle BmB' is isosceles and $Bm = B'm$.

$$R' - R = O'O = mB = \frac{B'r}{\text{vers } B'mB} = \frac{x'}{\text{vers } A}.$$

$$\therefore R' = R + \frac{x'}{\text{vers } A}. \quad \ldots\ldots \quad (14)$$

The solution is very similar in case the tangent is moved inward to $V''B''$. Note that this method necessarily changes the

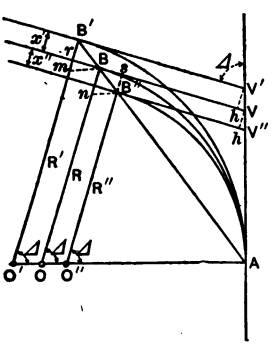


Fig. 20.

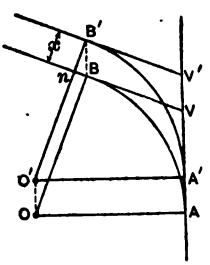


Fig. 21.

radius. If the radius is not to be changed, the point of curve must be altered as follows:

b. To move the forward tangent parallel to itself a distance x, the radius being unchanged. (Fig. 21.) In this case the whole

Digitized by Google

curve is moved bodily a distance $OO'=AA'=VV'=BB'$, and moved parallel to the first tangent AV

$$BB'=\frac{B'n}{\sin nBB'}=\frac{x}{\sin \Delta}=AA'. \quad . \quad . \quad . \quad (15)$$

c. To change the direction of the forward tangent at the point of tangency. (Fig. 22.) This problem involves a change (a) in the central angle and also requires a new radius. An error in the determination of the central angle furnishes an occasion for its use.

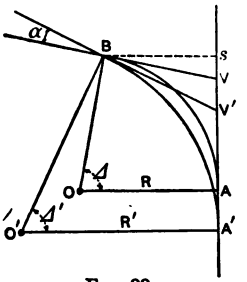


FIG. 22.

R, Δ, a, AV, and BV are known.

$$\Delta'=\Delta-a.$$

$$Bs=R \text{ vers } \Delta. \quad Bs=R' \text{ vers } \Delta'.$$

$$\therefore R'=R\frac{\text{vers } \Delta}{\text{vers } (\Delta-a)}. \quad . \quad (16)$$

$$As=R \sin \Delta. \quad A's=R' \sin \Delta'.$$

$$\therefore AA'=A's-As=R' \sin \Delta'-R \sin \Delta. \quad . \quad . \quad (17)$$

The above solutions are given to illustrate a large class of problems which are constantly arising. All of the ordinary problems can be solved by the application of elementary geometry and trigonometry.

34. Limitations in location. It may be required to run a curve that shall join two given tangents and also pass through a given point The point (P, Fig. 23) is assumed to be determined by its distance (VP) from the vertex and by the angle $AVP=\beta$.

It is required to determine the radius (R) and the tangent distance (AV). Δ is known.

$$PVG=\tfrac{1}{2}(180°-\Delta)-\beta$$
$$=90°-(\tfrac{1}{2}\Delta+\beta).$$
$$PP'=2VP \sin PVG$$
$$=2VP \cos (\tfrac{1}{2}\Delta+\beta).$$
$$PSV=\tfrac{1}{2}\Delta.$$
$$\therefore SP=VP\frac{\sin \beta}{\sin \tfrac{1}{2}\Delta}.$$

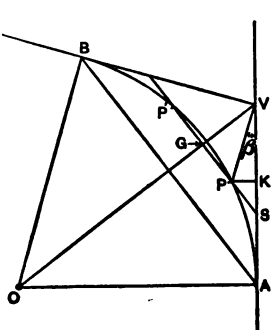


FIG. 23.

Digitized by Google

$$AS = \sqrt{SP \times SP'} = \sqrt{SP(SP + PP')}$$

$$= \sqrt{VP\frac{\sin\beta}{\sin\frac{1}{2}\Delta}\left[VP\frac{\sin\beta}{\sin\frac{1}{2}\Delta} + 2VP\cos(\tfrac{1}{2}\Delta + \beta)\right]}$$

$$= VP\sqrt{\frac{\sin^2\beta}{\sin^2\frac{1}{2}\Delta} + \frac{2\sin\beta\cos(\frac{1}{2}\Delta + \beta)}{\sin\frac{1}{2}\Delta}}.$$

$$SV = VP\frac{\sin(\frac{1}{2}\Delta + \beta)}{\sin\frac{1}{2}\Delta}.$$

$$AV = AS + SV$$

$$= \frac{VP}{\sin\frac{1}{2}\Delta}[\sin(\tfrac{1}{2}\Delta + \beta) + \sqrt{\sin^2\beta + 2\sin\beta\sin\tfrac{1}{2}\Delta\cos(\tfrac{1}{2}\Delta + \beta)}]. \quad (18)$$

$$R = AV\cot\tfrac{1}{2}\Delta.$$

In the special case in which P is on the median line OV, $\beta = 90° - \frac{1}{2}\Delta$, and $(\frac{1}{2}\Delta + \beta) = 90°$. Eq. 18 then reduces to

$$AV = \frac{VP}{\sin\frac{1}{2}\Delta}(1 + \cos\tfrac{1}{2}\Delta) = VP\cot\tfrac{1}{2}\Delta,$$

as might have been immediately derived from Eq. 8.

In case the point P is given by the offset PK and by the distance VK, the triangle PKV may be readily solved, giving the distance VP and the angle β, and the remainder of the solution will be as above.

35. Determination of the curvature of existing track. (a) *Using a transit.* Set up the transit at any point in the center of the track. Measure in each direction 100 feet to points also in the center of the track. Sight on one point with the plates at 0°. Plunge the telescope and sight at the other point. The angle between the chords equals the degree of curvature.

(b) *Using a tape and string.* Stretch a string (say 50 feet long) between two points on the inside of the head of the outer rail. Measure the ordinate (x) between the *middle* of the string and the head of the rail. Then

$$R = \frac{\text{chord}^2}{8x} \text{(very nearly)}. \quad . \quad . \quad . \quad . \quad (19)$$

For, in Fig. 24, since the triangles AOE and ADC are similar,

Digitized by Google

$AO : AE :: AD : DC$ or $R = \frac{1}{2}\overline{AD}^2 \div x$. When, as is usual, the arc is very short compared with the radius, $AD = \frac{1}{2}AB$, very nearly. Making this substitution we have Eq. 19. With a chord of 50 feet and a 10° curve, the resulting difference in x is .0025 of an inch—far within the possible accuracy of such a method. The above method gives the radius of the inner head of the outer rail. It should be diminished by $\frac{1}{2}g$ for the radius of the center of the track. With easy curvature, however, this will not affect the result by more than one or two tenths of one per cent.

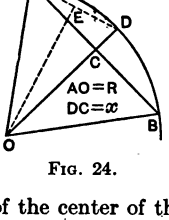


FIG. 24.

The inversion of this formula gives the required middle ordinate for a rail on a given curve. For example, the middle ordinate of a 30-foot rail, bent for a 6° curve, is

$$x = 900 \div (8 \times 955) = .118 \text{ foot} = 1.4 \text{ inches.}$$

Another much used rule is to require the foreman to have a string, knotted at the center, of such length that the middle ordinate, measured in inches, equals the degree of curve. To find that length, substitute (in Eq. 19) $5730 \div D$ for R and $D \div 12$ for x. Solving for *chord*, we obtain *chord* = 61.8 feet. The rule is not theoretically exact, but, considering the uncertain stretching of the string, the error is insignificant. In fact, the distance usually given is 62 feet, which is close enough for all purposes for which such a method should be used.

36. Problems. A systematic method of setting down the solution of a problem simplifies the work. Logarithms should always be used, and *all* the work should be so set down that a revision of the work to find a supposed error may be readily done. The value of such systematic work will become more apparent as the problems become more complicated. The two solutions given below will illustrate such work.

a. Given a 3° curve beginning at Sta. 27+60 and running to Sta. 32+45. Compute the ordinates and offsets used in locating the curve by tangential offsets.

b. With the same data as above, compute the distances to locate the curve by offsets from the long chord.

c. Assume that in Fig. 17 *ab* is measured as 217.6 feet, the

Digitized by Google

angle $abV = 17° 42'$, and the angle $baV = 21° 14'$. Join the tangents by a 4° 30′ curve. Determine bB and aA.

d. Assume that in a case similar to Fig. 18 it was noted that a distance (As) equal to 12 feet would clear the building. Assume that $\Delta = 38° 20'$ and that $D = 4° 40'$. Required the value of a and the position of n. *Solution:*

vers $a = As \div R$	$As = 12$	log = 1.07918
	R (for 4° 40′ curve)	log = 3.08923
	$a = 8° 01'$	log vers a = $\bar{7}$.98994
$ns = R \sin a$.		log sin a = $\bar{9}$.14445
		log R = 3.08923
	$ns = 171.27$	log = 2.23369

e. Assume that the forward tangent of a 3° 20′ curve having a central angle of 16° 50′ must be moved 3.62 feet *inward*, without altering the *P.C.* Required the change in radius.

f. Given two tangents making an angle of 36° 18′. It is required to pass a curve through a point 93.2 feet from the vertex, the line from the vertex to the point making an angle of 42° 21′ with the tangent. Required the radius and tangent distance *Solution:* Applying Eq. 18, we have

2	log = 0.30103
$\beta = 42° 21'$	log sin = $\bar{9}$.82844
$\frac{1}{2}\Delta = 18° 09'$	log sin = $\bar{9}$.49346
$(\frac{1}{2}\Delta + \beta) = 60° 30'$	log cos = $\bar{9}$.69234
.20667	$\bar{9}$.31527
log sin² β = 9.65688.... .45382	
2 | 9.81987.... .66049	
9.90993.... .81271	
nat. sin 60° 30′.... .8703	
1.6830....	log = 0.22610
$VP = 93.2$....	log = 1.96941
	2.19551
	log sin $\frac{1}{2}\Delta$ = $\bar{9}$.49346
Tang. dist. $AV = 503.36$....	log = 2.70205
	log cot $\frac{1}{2}\Delta$ = 10.48437
$R = 1536.1$....	log = 3.18642
$D = 3° 44'$	

Digitized by Google

COMPOUND CURVES.

37. Nature and use. Compound curves are formed by a succession of two or more simple curves of different curvature. The curves must have a common tangent at the point of compound curvature (*P.C.C.*). In mountainous regions there is frequently a necessity for compound curves having several changes of curvature. Such curves may be located separately as a succession of simple curves, but a combination of two simple curves has special properties which are worth investigating and utilizing. In the following demonstrations R_2 always represents the *longer* radius and R_1 the *shorter*, no matter which succeeds the other. T_1 is the tangent adjacent to the curve of shorter radius (R_1), and is invariably the shorter tangent. Δ_1 is the central angle of the curve of radius R_1, but it may be greater or less than Δ_2

38. Mutual relations of the parts of a compound curve having two branches. In Fig. 25, AC and CB are the two branches of

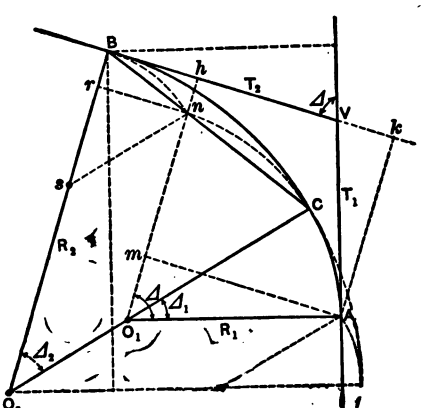


FIG. 25.

the compound curve having radii of R_1 and R_2 and central angles of Δ_1 and Δ_2. Produce the arc AC to n so that $AO_1n = \Delta$. The chord Cn produced *must* intersect B. The line ns, parallel to CO_2, will intersect BO_2 so that $Bs = sn = O_2O_1 = R_2 - R_1$. Draw Am perpendicular to O_1n. It will be parallel to hk.

Digitized by Google

$$Br = sn \text{ vers } Bsn \quad = (R_2 - R_1) \text{ vers } \Delta_2;$$
$$mn = AO_1 \text{ vers } AO_1n \quad = R_1 \text{ vers } \Delta;$$
$$Ak = AV \sin AVk \quad = T_1 \sin \Delta;$$
$$Ak = hm = mn + nh = mn + Br.$$
$$\therefore T_1 \sin \Delta = R_1 \text{ vers } \Delta + (R_2 - R_1) \text{ vers } \Delta_2. \quad . \quad . \quad (20)$$

Similarly it may be shown that

$$T_2 \sin \Delta = R_2 \text{ vers } \Delta - (R_2 - R_1) \text{ vers } \Delta_1. \quad . \quad . \quad (21)$$

The mutual relations of the elements of compound curves may be solved by these two equations. For example, assume the tangents as fixed (Δ therefore known) and that a curve of given radius R_1 shall start from a given point at a distance T_1 from the vertex, and that the curve shall continue through a given angle Δ_1. Required the other parts of the curve. From Eq. 20 we have

$$R_2 - R_1 = \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } \Delta_2}.$$

$$\therefore R_2 = R_1 + \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } (\Delta - \Delta_1)}. \quad . \quad . \quad . \quad . \quad . \quad (22)$$

T_2 may then be obtained from Eq. 21.

As another problem, given the location of the two tangents, with the two tangent distances (thereby locating the *PC* and *PT*), and the central angle of each curve; required the two radii. Solving Eq. 20 for R_1, we have

$$R_1 = \frac{T_1 \sin \Delta - R_2 \text{ vers } \Delta_2}{\text{vers } \Delta - \text{vers } \Delta_2}.$$

Similarly from Eq. 21 we may derive

$$R_1 = \frac{T_2 \sin \Delta - R_2(\text{vers } \Delta - \text{vers } \Delta_1)}{\text{vers } \Delta_1}.$$

Equating these, reducing, and solving for R_2, we have

$$R_2 = \frac{T_1 \sin \Delta \text{ vers } \Delta_1 - T_2 \sin \Delta \text{ (vers } \Delta - \text{vers } \Delta_2)}{\text{vers } \Delta_2 \text{ vers } \Delta_1 - (\text{vers } \Delta - \text{vers } \Delta_1)(\text{vers } \Delta - \text{vers } \Delta_2)}. \quad (23)$$

Although the various elements may be chosen as above with considerable freedom, there are limitations. For example, in Eq. 22, since R_2 is always greater than R_1, the term to be added to R_1 must be essentially positive—i.e., $T_1 \sin \Delta$ must be greater than R_1 vers Δ. This means that $T_1 > R_1 \frac{\text{vers } \Delta}{\sin \Delta}$, or that

Digitized by Google

$T_1 > R_1 \tan \frac{1}{2}\Delta$, or that T_1 is greater than the corresponding tangent on a simple curve. Similarly it may be shown that T_2 is *less* than $R_2 \tan \frac{1}{2}\Delta$ or *less* than the corresponding tangent on a simple curve. Nevertheless T_2 is always greater than T_1. In the limiting case when $R_2 = R_1$, $T_2 = T_1$, and $\Delta_2 = \Delta_1$.

39. Modifications of location. Some of these modifications may be solved by the methods used for simple curves. For example·

a. It is desired to move the tangent VB, Fig. 26, parallel to itself to $V'B'$. Run a new curve from the *P.C.C.* which shall reach the new tangent at B', where the chord of the old curve

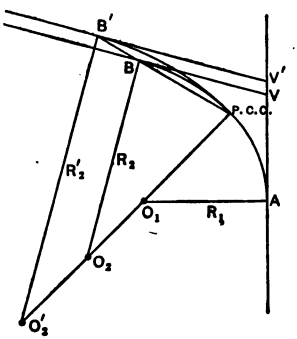


FIG. 26.

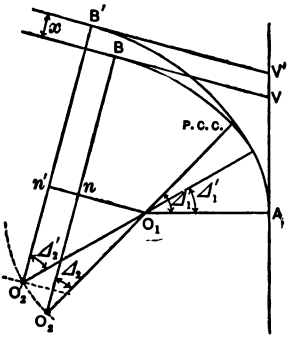


FIG. 27.

intersects the new tangent. The solution is almost identical with that in § 33, *a*.

b. Assume that it is desired to change the forward tangent (as above) but to retain the same radius. In Fig. 27

$$(R_2 - R_1)\cos \Delta_2 = O_2 n;$$
$$(R_2 - R_1)\cos \Delta_2' = O_2' n'.$$
$$x = O_2 n - O_2' n' = (R_2 - R_1)(\cos \Delta_2 - \cos \Delta_2').$$

$$\cos \Delta_2' = \cos \Delta_2 - \frac{x}{R_2 - R_1}. \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The *P.C.C.* is moved *backward* along the sharper curve an angular distance of $\Delta_2' - \Delta_2 = \Delta_1 - \Delta_1'$.

In case the tangent is moved inward rather than outward, the solution will apply by transposing Δ_2 and Δ_2'. Then we shall have

$$\cos \Delta_2' = \cos \Delta_2 + \frac{x}{R_2 - R_1}. \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Digitized by Google

The *P.C.C.* is then moved *forward*.

c. Assume the same case as (b) except that the larger radius comes first and that the tangent adjacent to the smaller radius is moved. In Fig. 28

$$(R_2-R_1)\cos \Delta_1 = O_1n;$$

$$(R_2-R_1)\cos \Delta_1' = O_1'n'.$$

$$x = O_1'n' - O_1n$$
$$= (R_2-R_1)(\cos \Delta_1' - \cos \Delta_1).$$

$$\cos \Delta_1' = \cos \Delta_1 + \frac{x}{R_2-R_1}. \quad (26)$$

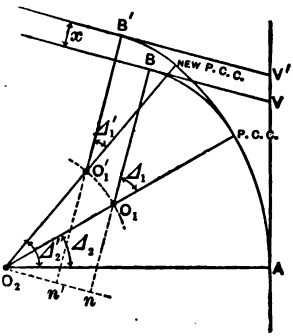


Fig. 28.

The *P.C.C.* is moved *forward* along the easier curve an angular distance of $\Delta_1' - \Delta_1 = \Delta_2 - \Delta_2'$.

In case the tangent is moved *inward*, transpose as before and we have

$$\cos \Delta_1' = \cos \Delta_1 - \frac{x}{R_2-R_1}. \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The *P.C.C.* is moved *backward*

d. Assume that the radius of one curve is to be altered without changing either tangent. Assume conditions as in Fig. 29. For the diagrammatic solution assume that R_2 is to be increased by O_2S. Then, since R_2' must pass through O_1 and extend beyond O_1 a distance O_1S, the locus of the new center must lie on the arc drawn about O_1 as center and with OS as radius. The locus of O_2' is also given by a line $O_2'p$ parallel to BV and at a distance of R_2' (equal to $S \ldots P.C.C.$) from it. The new center is therefore at the intersection O_2'. An arc with radius R_2' will therefore be tangent at B' and tangent to the old curve *produced* at NEW *P.C.C.* Draw O_1n' perpendicular to O_2B.

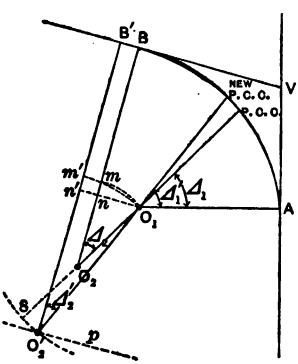


Fig. 29.

Digitized by Google

With O_2 as center draw the arc O_1m, and with O_2' as center draw the arc O_1m'. $mB=m'B'=R_1$.

$$\therefore\ mn=m'n'=(R_2'-R_1)\ \text{vers}\ \Delta_2'=(R_2-R_1)\ \text{vers}\ \Delta_2.$$

$$\therefore\ \text{vers}\ \Delta_2'=\frac{(R_2-R_1)}{(R_2'-R_1)}\ \text{vers}\ \Delta_2. \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$O_1n=(R_2-R_1)\sin\Delta_2;$$

$$O_1n'=(R_2'-R_1)\sin\Delta_2'.$$

$$BB'=O_1n'-O_1n=(R_2'-R_1)\sin\Delta_2'-(R_2-R_1)\sin\Delta_2. \quad (29,$$

This problem may be further modified by assuming that the radius of the curve is decreased rather than increased, or that the smaller radius follows the larger. The solution is similar and is suggested as a profitable exercise.

It might also be assumed that, instead of making a given change in the radius R_2, a given change BB' is to be made. Δ_2' and R_2' are required. Eliminate R_2' from Eqs. 28 and 29 and solve the resulting equation for Δ_2'. Then determine R_2' by a suitable inversion of either Eq. 28 or 29.

As in §§ 32 and 33, the above problems are but a few, although perhaps the most common, of the problems the engineer may meet with in compound curves. All of the ordinary problems may be solved by these and similar methods.

40. Problems. *a.* Assume that the two tangents of a compound curve are to be 348 feet and 624 feet, and that $\Delta_1=22°\ 16'$ and $\Delta_2=28°\ 20'$. Required the radii.

[*Ans.* $R_1=326.92$; $R_2=1574.85$.]

b. A line crosses a valley by a compound curve which is first a 6° curve for 46° 30′ and then a 9° 30′ curve for 84° 16′. It is afterward decided that the last tangent should be 6 feet farther up the hill. What are the required changes? [*Note.* The second tangent is evidently moved *outward*. The solution corresponds to that in the first part of § 39, *c*. The *P.C.C.* is moved forward 16.39 feet. If it is desired to know how far the *P.T.* is moved in the direction of the tangent (i.e., the *projection* of BB', Fig. 28, on $V'B'$), it may be found by observing that it is equal to $nn'=(R_2-R_1)(\sin\Delta_1-\sin\Delta_1')$. In this case it equals 0.65 foot, which is very small because Δ_1 is nearly 90°. The value of Δ_2 (46° 30′) is not used, since the solution is independent of the value of Δ_2. The student should learn to recognize

Digitized by Google

which quantities are mutually related and therefore essential to a solution, and which are independent and non-essential.]

TRANSITION CURVES. → 47

41. Superelevation of the outer rail on curves. When a mass is moved in a circular path it requires a centripetal force to keep it moving in that path. By the principles of mechanics we know that this force equals $Gv^2 \div gR$, in which G is the weight, v the velocity in feet per second, g the acceleration of gravity in feet per second in a second, and R the radius of curvature. If the two rails of a curved track were laid on a level (transversely), this centripetal force could only be furnished by the pressure of the wheel-flanges against the rails. As this is very objectionable, the outer rail is elevated so that the reaction of the rails against the wheels shall contain a horizontal component equal to the required centripetal force. In Fig. 30, if *ob* represents the reaction, *oc* will represent the weight G, and *ao* will represent the required centripetal force. From similar triangles we may write $sn : sm :: ao : oc$. Call $g = 32.17$. Call $R = 5730 \div D$, which is sufficiently accurate for this purpose (see § 19). Call $v = 5280V \div 3600$, in which V is the velocity in miles per hour. *mn* is the distance between rail centers, which, for an 80-lb. rail and standard gauge, is 4.916 feet *sm* is slightly less than this. As an average value we may call it 4.900, which is its exact value when the superelevation is $4\frac{3}{4}$ inches. Calling $sn = e$, measured in feet, we have

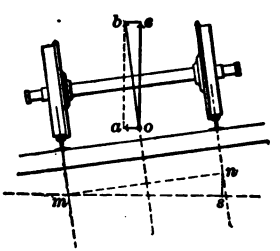


FIG. 30.

$$e = sm\frac{ao}{oc} = 4.9\frac{Gv^2}{gR}\frac{1}{G} = \frac{4.9 \times 5280^2 V^2 D}{32.17 \times 3600^2 \times 5730}.$$

$$e = .0000572V^2D. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

It should be noticed that, according to this formula, the required superelevation varies as the *square* of the velocity, which means that a change of velocity of only 10% would call for a change of superelevation of 21%. Since the velocities of trains over any road are extremely variable, it is impossible to adopt

Digitized by Google

any superelevation which will fit all velocities even approximately. The above fact also shows why any over-refinement in the calculations is useless and why the above approximations, which are really small, are amply justifiable. For example, the above formula contains the approximation that $R = 5730 \div D$. In the extreme case of a 10° curve the error involved would be about 1%. A change of about ½ of 1% in the velocity, or say from 40 to 40.2 miles per hour, would mean as much. The error in e due to the assumed constant value of sm is never more than a very small fraction of 1%. The rail-laying is not done closer than this. The following tabular form is based on Eq. (30):

SUPERELEVATION OF THE OUTER RAIL (IN FEET) FOR VARIOUS VELOCITIES AND DEGREES OF CURVATURE.

Velocity in Miles per Hour.	Degree of Curve.									
	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
30	.05	.10	.15	.20	.26	.31	.36	.41	.46	.51
40	.09	.18	.27	.37	.46	.55	.64	.73	.82	
50	.14	.29	.43	.57	.71	.86				
60	.20	.41	.62	.82						

42. Practical rules for superelevation. A much used rule for superelevation is to "elevate one half an inch for each degree of curvature." The rule is rational in that e in Eq. 30 varies directly as D. The above rule therefore agrees with Eq. 30 when V is about 27 miles per hour. However applicable the rule may have been in the days of low velocities, the elevation thus computed is too small now. The rule to elevate one inch for each degree of curvature is also used and is precisely similar in its nature to the above rule. It agrees with Eq. 30 when the velocity is about 38 miles per hour, which is more nearly the average speed of trains.

Another (and better) rule is to "elevate for the speed of the fastest trains." This rule is further justified by the fact that a four-wheeled truck, having two parallel axles, will always tend to run to the outer rail and will require considerable flange pressure to guide it along the curve. The effect of an excess of superelevation on the slower trains will only be to relieve this flange pressure somewhat. This rule is coupled with the limitation

Digitized by Google

that the elevation should never exceed a limit of six inches—sometimes eight inches. This limitation implies that locomotive engineers must reduce the speed of fast trains around sharp curves until the speed does not exceed that for which the actual superelevation used is suitable. The heavy line in the tabular form (§ 41) shows the six-inch limitation.

Some roads furnish their track foremen with a list of the superelevations to be used on each curve in their sections. This method has the advantage that each location may be separately studied, and the proper velocity, as affected by local conditions (*e.g.*, proximity to a stopping-place for all trains), may be determined and applied.

Another method is to allow the foremen to determine the superelevation for each curve by a simple measurement taken at the curve. The rule is developed as follows: By an inversion of Eq. 19 we have

$$x = chord^2 \div 8R. \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Putting x equal to e in Eq. 30 and solving for "*chord*," we have

$$chord^2 = .0000572V^2D\mathcal{E}R$$
$$= 2.621V^2.$$
$$chord = 1.62V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

To apply the rule, assume that 50 miles per hour is fixed as the velocity from which the superelevation is to be computed. Then $1.62V = 1.62 \times 50 = 81$ feet, which is the distance given to the trackmen. Stretch a tape (or even a string) with a length of 81 feet between two points on the concave side of the head of either the inner or the outer rail. The ordinate at the middle point then equals the superelevation. The values of this chord length for varying velocities are given in the accompanying tabular form.

Velocity in miles per hour...	20	25	30	35	40	45	50	55	60
Chord length in feet.....	32.4	40.5	48.6	56.7	64.8	72.9	81.0	89.1	97.2

The following tabular form shows the standard (at one time) on the N. Y., N. H. & H. R. R. It should be noted that the elevations do not increase proportionately with the radius, and that they are higher for descending grades than for level or

Digitized by Google

ascending grades. This is on the basis that the velocity on curves and on ascending grades will be less than on descending grades. For example, the superelevation for a 0° 30′ curve on a descending grade corresponds to a velocity of about 54 miles per hour, while for a 4° curve on a level or ascending grade the superelevation corresponds to a velocity of only about 38 miles per hour.

TABLE OF THE SUPERELEVATION OF THE OUTER RAIL ON CURVES, N. Y., N. H. & H. R. R.

Degree of curve.	Level or ascending grade.	Descending grade.
	inches.	inches.
0° 30′	0¾	1
1 00	1¼	1¾
1 15	1¾	2
1 30	2	2¼
1 45	2¼	2½
2 00	2½	2¾
2 15	2¾	3
2 30	2¾	3¼
2 45	3	3½
3 00	3¼	3¾
3 15	3½	3¾
3 30	3¾	4
3 45	3¾	4¼
4 00	4	4½

43. Transition from level to inclined track. On curves the track is inclined transversely; on tangents it is level. The transition from one condition to the other must be made gradually. If there is no transition curve, there must be either inclined track on the tangent or insufficiently inclined track on the curve or both. Sometimes the full superelevation is continued through the total length of the curve and the "run-off" (having a length of 100 to 400 feet) is located entirely on the tangents at each end. In other practice it is located partly on the tangent and partly on the curve. Whatever the method, the superelevation is correct at only one point of the run-off. At all other points it is too great or too small. This (and other causes) produces objectionable lurches and resistances when entering and leaving curves. The object of transition curves is to obviate these resistances.

On the Lehigh Valley R. R. the run-off is made in the form of a reversed vertical curve, as shown in the accompanying figure. According to this system the length of run-off varies

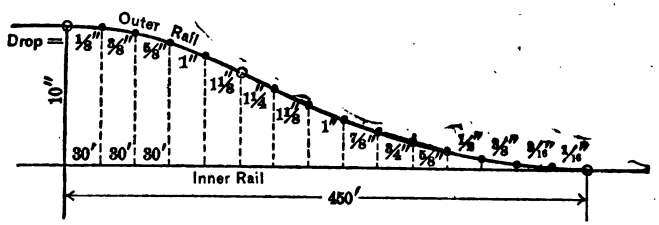
Digitized by Google

from 120 feet, for a superelevation of one inch, to 450 feet, for a superelevation of ten inches. Such a superelevation as ten inches is very unusual practice, but is successfully operated on that road. The curve is concave upward for two-thirds of its length and then reverses so that it is convex upward.

TABLE FOR RUN-OFF OF ELEVATION OF OUTER RAIL OF CURVES.

Drop in inches for each 30-foot rail commencing at theoretical point of curve.

Elevation.	1/8″	1/4″	3/8″	1/2″	5/8″	3/4″	7/8″	1″	1 1/8″	1 1/4″	1 3/8″	1 1/2″	1 5/8″	1 3/4″	1 7/8″	2″	2 1/16″	2 1/8″	2 3/16″	Total.
1″	..	30	30	..	..	..	..	..	..	..	..	..	..	..	..	..	30	..	30	120
2″	..	30	..	..	..	30	..	..	..	..	..	..	..	..	30	30	..	..	30	150
3″	..	30	..	..	..	30	..	..	..	..	..	..	30	..	30	..	30	..	30	180
4″	..	30	..	30	..	..	30	..	..	..	..	..	30	..	30	30	..	30	30	240
5″	..	30	..	30	..	..	..	30	..	..	..	..	30	..	30	30	30	..	30	270
6″	..	30	..	30	..	30	..	..	30	..	..	..	30	..	30	30	30	..	30	300
7″	..	30	..	30	..	30	..	30	..	..	30	30	..	30	30	..	30	30	..	330
8″	..	30	..	30	..	30	..	30	30	30	30	..	30	30	..	30	..	30	30	360
9″	30	..	..	30	..	30	30	30	..	30	30	30	30	30	30	..	30	..	30	420
10″	30	..	30	..	30	..	..	30	30	30	30	30	30	30	30	..	30	..	30	450



The figure (and also the lower line of the tabulated form) shows the drop for each thirty-foot rail length. For shorter lengths of run-off, the drop for each 30 feet is shown by the corresponding lines in the tabular form. Note in each horizontal line that the sum of the drops, under which 30 is found, equals the total superelevation as found in the first column. For example, for 4 inches superelevation, length of curve 240 feet, the successive drops are $\frac{1}{4}''$, $\frac{1}{2}''$, $\frac{7}{8}''$, $\frac{7}{8}''$, $\frac{5}{8}''$, $\frac{1}{2}''$, $\frac{1}{4}''$, and $\frac{1}{8}''$ whose sum is 4 inches. Possibly the more convenient form would be to indicate for each 30-foot point the actual superelevation of the outer rail, which would be for the above case (running from the tangent to the curve) $\frac{1}{8}''$, $\frac{3}{8}''$, $\frac{7}{8}''$, $1\frac{1}{2}''$, $2\frac{3}{8}''$, $3\frac{1}{4}''$, $3\frac{3}{4}''$, $4''$.

44. Fundamental principle of transition curves. If a curve

Digitized by Google

has variable curvature, beginning at the tangent with a curve of infinite radius, and the curvature gradually sharpens until it equals the curvature of the required simple curve and there becomes tangent to it, the superelevation of such a transition curve may begin at zero at the tangent, gradually increase to the required superelevation for the simple curve, and yet have at every point the superelevation required by the curvature at that point. Since in Eq. (30) e is directly proportional to D, the required curve must be one in which the degree of curve increases directly as the distance along the curve. The mathematical development of such a curve is quite complicated. It has, however, been developed, and tables have been computed for its use, by Prof. C. L. Crandall. The following method has the advantage of great simplicity, while its agreement with the true transition curve is as close as need be, as will be shown.

45. Multiform compound curves. If the transition curve commences with a very flat curve and at regular even chord lengths compounds into a curve of sharper curvature until the desired curvature is reached, the increase in curvature at each chord point being uniform, it is plain that such a curve is a close approximation to the true spiral, especially since the rails as laid will *gradually* change their curvature rather than maintain a uniform curvature throughout each chord length and then abruptly change the curvature at the chord points. Such a curve, *as actually laid*, will be a much closer approximation to the true curve than the multiform compound curve by which it is set out. There will actually be a *gradual* increase in curvature which increases directly as the length of the curve.

46. Required length of spiral. The required length of spiral evidently depends on the amount of superelevation to be gained, and also depends somewhat on the speed. If the spiral is laid off in 25-foot chord lengths, with the first chord subtending a 1° curve, the second a 2° curve, etc., the fifth chord will subtend a 5° curve, and the increase from this last chord to a 6° curve is the same as the uniform increase of curvature between the chords. The same spiral extended would run on to a 12° curve in $(12-1)25=275$ feet. The last chord of a spiral should have a smaller degree of curvature than the simple curve to which it is joined. If the curves are very sharp, such as are used in street work and even in suburban trolley work, an increase in degree of curvature of 1° per 25 feet will not be sufficiently rapid, as

Digitized by Google

such a rate would require too long curves. 2°, 10°, or even 20° increase per 25 feet may be necessary, but then the chords should be reduced to 5 feet. Such a rapid rate of increase is justified by the necessary reduction in speed. On the other hand, very high speed will make a lower rate of increase desirable, and therefore a spiral whose degree of curvature increases only 0° 30′ per 25 feet may be used. Such a spiral would require a length of 375 feet to run on to an 8° curve, which is inconveniently long, but it might be used to run on to a 4° curve, where its length would be only 175 feet. Three spirals have been developed in Table IV, each with chords of 25 feet, the rate of increase in the degree of curvature being 0° 30′, 1° and 2° per chord. One of these will be suitable for any curvature found on ordinary steam-railroads.

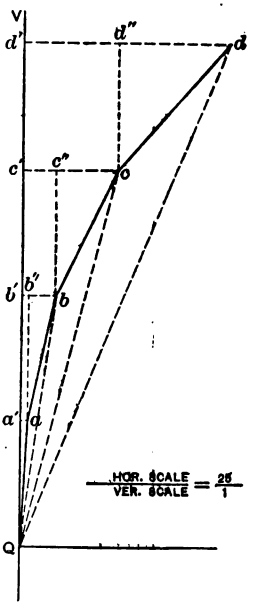


FIG. 31.

47. To find the ordinates of a 1°-per-25-feet spiral. Since the first chord subtends a 1° curve, its central angle is 0° 15′ and the angle aQV (Fig. 31) is 7′ 30″. The tangent at a makes an angle of 15′ with VQ. The angle between the chord ba and the tangent at a is $\frac{1}{2}(30') = 15'$, and the angle $bab'' = \frac{1}{2}(30') + 15' = 30'$. Similarly

the angle $cbc'' = \frac{1}{2}(45') + 30' + 15' = 67'\ 30'' = 1°\ 07'\ 30''$,

and the angle $dcd'' = 2°\ 0'$.

The ordinate $aa' = 25 \sin 7'\ 30''$, and

$$Qa' = 25 \cos 7'\ 30''.$$

$$Qb' = Qa' + a'b'$$
$$= Qa' + ab''$$
$$= 25 (\cos 7'\ 30'' + \cos 30').$$

$$bb' = b'b'' + bb''$$
$$= 25 (\sin 7'\ 30'' + \sin 30').$$

Similarly the ordinates of c, d, etc., may be obtained.

Digitized by Google

48. To find the deflections from any point of the spiral. $aQV = 7'\ 30''$. Tan $bQV = bb' \div Qb'$; tan $cQV = cc' \div Qc'$; etc. Thus we are enabled to find the deflection angles from the tangent at Q to any point of the spiral.

The tangent to the curve at c (Fig. 32) makes an angle of

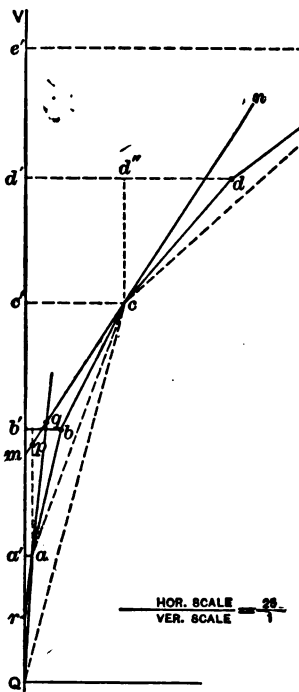


FIG. 32.

$1° 30'$ with QV, or $cmV = 1° 30'$. $Qcm = cmV - cQm$. The value of cQm is known from previous work. The deflection from c to Q then becomes known.

$acm = cmV - cap = cmV - caq - qap$. caq is the deflection angle to c from the tangent at a and will have been previously computed numerically. $qap = 15'$. acm therefore becomes known.

$$bcm = \tfrac{1}{2} \text{ of } 45' = 22'\ 30'';$$
$$dcn = \tfrac{1}{2} \text{ of } 60' = 30'.$$

Digitized by Google

$ecn = ecd'' - ncd''$, $ncd'' = cmV$, tan $ecd'' = (ee' - d''d') \div c'e'$, all of which are known from the previous work.

By this method the deflections from the tangent at any point

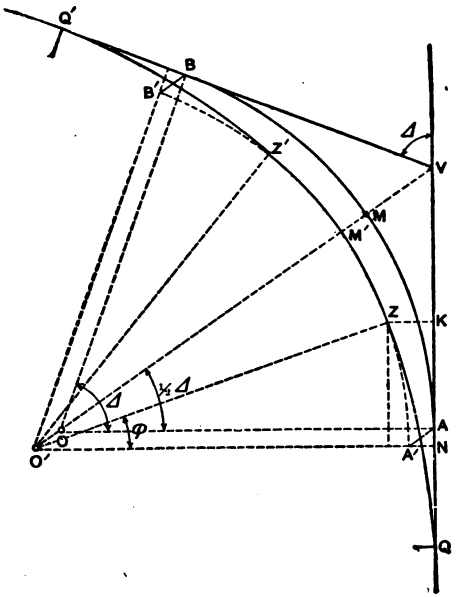


FIG. 33.

of the curve to any other point are determinable. These values are compiled in Table IV. The corresponding values of these angles when the increase in the degree of curvature per chord length is 30′, and when it is 2°, are also given in Table IV.

49. Connection of spiral with circular curve and with tangent. See Fig. 33.* Let AV and BV be the tangents to be connected

* The student should at once appreciate the fact of the necessary distortion of the figure. The distance MM' in Fig. 33 is perhaps 100 times its real proportional value.

Digitized by Google

by a $D°$ curve, having a suitable spiral at each end. If no spirals were to be used, the problem would be solved as in simple curves giving the curve AMB Introducing the spiral has the effect of throwing the curve away from the vertex a distance MM' and reducing the central angle of the $D°$ curve by 2ϕ. Continuing the curve beyond Z and Z' to A' and B', we will have $AA'=BB'=MM'$. $ZK=$the x ordinate and is therefore known. Call $MM'=m$. $A'N=x-R$ vers ϕ. Then

$$m=MM'=AA'=\frac{A'N}{\cos\frac{1}{2}\Delta}=\frac{x-R\text{ vers }\phi}{\cos\frac{1}{2}\Delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

$$\begin{aligned} NA &= AA'\sin\tfrac{1}{2}\Delta=(x-R\text{ vers }\phi)\tan\tfrac{1}{2}\Delta, \\ VQ &= QK-KN+NA+AV \\ &= y-R\sin\phi+(x-R\text{ vers }\phi)\tan\tfrac{1}{2}\Delta+R\tan\tfrac{1}{2}\Delta \\ &= y-R\sin\phi+x\tan\tfrac{1}{2}\Delta+R\cos\phi\tan\tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad . \quad (34) \end{aligned}$$

When $A'N$ has already been computed, it may be more convenient to write

$$VQ=y+R(\tan\tfrac{1}{2}\Delta-\sin\phi)+A'N\tan\tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad . \quad (35)$$

$$\begin{aligned} VM' &= VM+MM' \\ &= R\text{ exsec }\tfrac{1}{2}\Delta+\frac{x}{\cos\frac{1}{2}\Delta}-\frac{R\text{ vers }\phi}{\cos\frac{1}{2}\Delta}. \quad . \quad . \quad . \quad . \quad . \quad (36) \end{aligned}$$

$$\begin{aligned} AQ &= VQ-AV \\ &= y-R\sin\phi+(x-R\text{ vers }\phi)\tan\tfrac{1}{2}\Delta. \quad . \quad . \quad . \quad . \quad (37) \end{aligned}$$

A method of obtaining the necessary dimensions using tables, is given in § 53*a*.

Example. To join two tangents making an angle of 34° 20′ by a 5° 40′ curve and suitable spirals. Use 1°-per-25-feet spirals with five chords. Then $\phi=3°\ 45'$, $x=2.999$, $\frac{1}{2}\Delta=17°\ 10'$, and $y=124.942$.

Digitized by Google

[Eq. 33]		R	3.00497
		vers ϕ	7.33063
	2.166		$\overline{0.33560}$
	$x=$ 2.999		
	$A'N=$ 0.833		$9.92064\overline{1}$
		cos ½D	9.98021
	$m=MM'=AA'$ = 0.872		$9.94043\overline{5}$
[Eq. 36]		R	3.00497
		exsec ½D	$8.66863\overline{5}$
	$VM=47.164$		$1.67360\overline{0}$
	$m=$ 0.872		
	$VM'=48.036$		
[Eq. 35]	$y=124.942$ nat. tan ½D = .30891		
	nat. sin $\phi=$ .06540		
	.24351		$9.38651\overline{1}$
		R	3.00497
	246.314		$2.39148\overline{5}$
	[See above]	$A'N$	9.92064
		tan ½D	9.48984
	0.257	AN	$9.41048\overline{5}$
	$VQ=371.513$		
[Eq. 37]		R	3.00497
		tan ½D	9.48984
	312.471	AV	2.49481
	$AQ=$ 59.042		

50. **Field-work.** When the spiral is designed during the original location, the tangent distance VQ should be computed and the point Q located. It is hardly necessary to locate all of the points of the spiral until the track is to be laid. The extremities should be located, and as there will usually be one and perhaps two full station points on the spiral, these should also be located. Z may be located by setting off $QK=y$ and $KZ=x$, or else by the tabular deflection for Z from Q and the distance ZQ, which is the long chord. Setting up the instrument at Z and sighting back at Q with the proper deflection, the tangent at Z may be found and the circular curve located as usual, its central angle being $D-2\phi$. A similar operation will locate Q' from Z'.

To locate points on the spiral. Set up at Q, with the plates

Digitized by Google

reading 0° when the telescope sights along VQ. Set off from Q the deflections given in Table IV for the instrument at Q, using a chord length of 25 feet, the process being like the method for simple curves except that the deflections are irregular. If a full station-point occurs within the spiral, interpolate between the deflections for the adjacent spiral-points. For example, a spiral begins at Sta. 56+15. Sta. 57 comes 10 feet beyond the third spiral point. The deflection for the third point is 35′ 0″; for the fourth it is 56′ 15″. $\frac{1}{2}\frac{0}{5}$ of the difference (21′ 15″) is 8′ 30″; the deflection for Sta. 57 is therefore 43′ 30″. This method is not theoretically accurate, but the error is small. Arriving at Z, the forward alignment may be obtained by sighting back at Q (or at any other point) with the given deflection

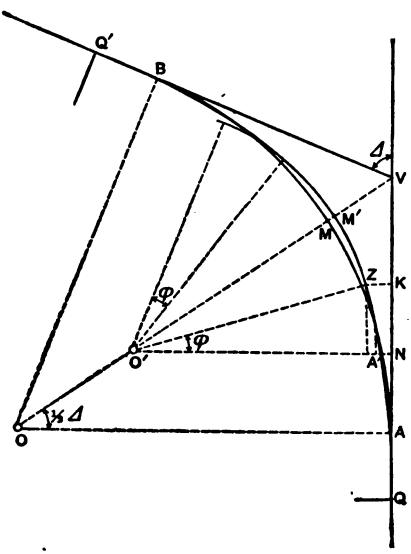


FIG. 34.

for that point from the station occupied. Then when the plates read 0° the telescope will be tangent to the spiral and to the succeeding curve. All rear points should be checked from Z. If it is necessary to occupy an intermediate station, use the deflections given for that station, orienting as just explained for Z,

Digitized by Google

checking the back points and locating all forward points up to Z if possible.

After the center curve has been located and Z' is reached, the other spiral must be located but *in reverse order*, i.e., the sharp curvature of the spiral is at Z' and the curvature decreases toward Q'.

51. To replace a simple curve by a curve with spirals. This *may* be done by the method of § 49, but it involves shifting the whole track a distance m, which in the given example equals 0.87 foot. Besides this the track is appreciably shortened, which would require rail-cutting. But the track may be kept at practically the same length and the lateral deviation from the old track may be made very small by slightly sharpening the curvature of the old track, moving the new curve so that it is wholly or partially *outside* of the old curve, the remainder of it with the spirals being *inside* of the old curve. It is found by experience that a decrease in radius of from 1% to 5% will answer the purpose. The larger the central angle the less the change. The solution is as indicated in Fig. 34.

$$O'N = R' \cos \phi + x.$$

$$O'V = O'N \sec \tfrac{1}{2}\Delta$$
$$= R' \cos \phi \sec \tfrac{1}{2}\Delta + x \sec \tfrac{1}{2}\Delta.$$

$$m = MM' = MV - M'V$$
$$= R \operatorname{exsec} \tfrac{1}{2}\Delta - (O'V - R')$$
$$= R \operatorname{exsec} \tfrac{1}{2}\Delta - R' \cos \phi \sec \tfrac{1}{2}\Delta - x \sec \tfrac{1}{2}\Delta + R'. \quad . \quad . \quad . \quad (38)$$

$$AQ = QK - KN + NV - VA$$
$$= y - R' \sin \phi + (R' \cos \phi + x) \tan \tfrac{1}{2}\Delta - R \tan \tfrac{1}{2}\Delta$$
$$= y - R' \sin \phi + R' \cos \phi \tan \tfrac{1}{2}\Delta - (R - x) \tan \tfrac{1}{2}\Delta. \quad . \quad . \quad (39)$$

The length of the old curve from Q to $Q' = 2AQ + 100\frac{\Delta}{D}$.

The length of the new curve from Q to $Q' = 2L + 100\frac{\Delta - 2\phi}{D'}$,

in which L is the length of each spiral.

Example. Suppose the old curve is a 7° 30′ curve with a central angle of 38° 40′. As a trial, compute the relative length of a new 8° curve with spirals of seven chords. $\phi = 7°\ 0'$; $\frac{1}{2}\Delta = 19°\ 20'$; R (for the 7° 30′ curve) = 764.489; R' (for the 8° curve) = 716.779; $x = 7.628$.

Digitized by Google

[Eq. 38]

		R exsec ½Δ	2.88337
			8.77642
45.687			1.65970
R' = 716.779			
762.466		R' cos ϕ	2.85538
		sec ½Δ	0.09675
			0.02521
	753.953		2.87734
		x sec ½Δ	0.88241
	c		0.02521
	8.084		0.90762
762.087	762.037		
m = 0.429			

[Eq. 39]

y = 174.722		R' sin φ	2.85538
			9.08580
	87.353		1.94128
		R' cos φ	2.85538
		tan ½Δ	9.99675
			9.54512
249.606			2.39725
		R = 764.489	
		x = 7.628	
		756.861 tan ½Δ	2.87901
			9.54512
	265.543		2.42413
424.328	352.896		
352.896			
AQ = 71.432			

The length of the old curve from Q to Q' is

$100\,\frac{\Delta}{D} = 100\,\frac{38.667}{7.5}$ =	515.556
$2AQ = 2 \times 71.432$ =	142.864
	658.420
New curve: $100\,\frac{\Delta - 2\phi}{D'} = 100\,\frac{38.667 - 14.000}{8.0}$ = 308.333	
$2L = 2 \times 175$ = 350.000	
658.333	658.333
Difference in length =	0.087

Considering that this difference may be divided among 22 joints (using 30-foot rails) no rail-cutting would be necessary. If the difference is too large, a slight variation in the value of the new radius R' will reduce the difference as much as necessary. A truer comparison of the lengths would be found by comparing the lengths of the arcs.

52. Application of transition curves to compound curves. Since compound curves are only employed when the location is limited by local conditions, the elements of the compound curve should be determined (as in §§ 38 and 39) regardless of the

Digitized by Google

transition curves, depending on the fact that the lateral shifting of the curve when transition curves are introduced is very small. If the limitations are very close, an estimated allowance may be made for them.

Methods have been devised for inserting transition curves between the branches of a compound curve, but the device is

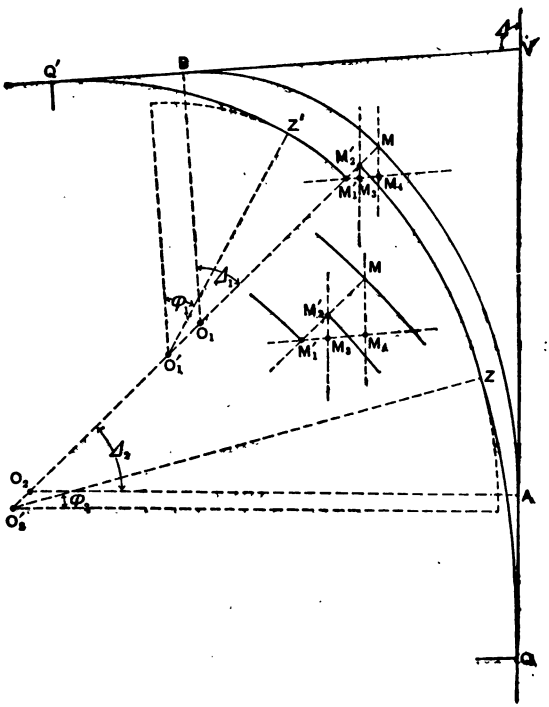


FIG. 35.

complicated and usually needless, since when the train is once on a curve the wheels press against the outer rail steadily and a change in curvature will not produce a serious jar even though the superelevation is temporarily a little more or less than it should be.

Digitized by Google

If the easier curve of the compound curve is less than 3° or 4°, there may be no need for a transition curve off from that branch. This problem then has two cases according as transition curves are used at both ends or at one end only.

a. *With transition curves at both ends.* Adopting the method of § 49, calling $\Delta_1 = \frac{1}{2}\Delta$, we may compute $m_1 = MM_1'$. Similarly, calling $\Delta_2 = \frac{1}{2}\Delta$, we may compute $m_2 = MM_2'$. But M_1' and M_2' must be made to coincide. This may be done by moving the curve $Z'M_1'$ and its transition curve parallel to $Q'V$ a distance $M_1'M_3$, and the other curve parallel to QV a distance $M_2'M_3$. In the triangle $M_1'M_3M_2'$, the angle at $M_1' = 90° - \Delta_1$, the angle at $M_2' = 90° - \Delta_2$, and the angle at $M_3 = \Delta$.

$$\left.\begin{aligned} &\text{Then } M_1'M_3 = M_1'M_2' \frac{\sin (90° - \Delta_2)}{\sin \Delta} = (m_1 - m_2) \frac{\cos \Delta_2}{\sin \Delta}. \\ &\text{Similarly } M_2'M_3 = M_1'M_2' \frac{\sin (90° - \Delta_1)}{\sin \Delta} = (m_1 - m_2) \frac{\cos \Delta_1}{\sin \Delta}. \end{aligned}\right\} \quad (40)$$

b. *With a transition curve on the sharper curve only.* Compute $m_1 = MM_1'$ as before; then move the curve Z_1M_1' parallel to $Q'V$ a distance of

$$M_1'M_4 = m_1 \frac{\cos \Delta_2}{\sin \Delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

The simple curve MA is moved parallel to VA a distance of

$$MM_4 = m_1 \frac{\cos \Delta_1}{\sin \Delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

If Δ_1 and Δ_2 are both small, $M_1'M_4$ and MM_4 may be more than m_1, but the lateral deviation of the new curve from the old will always be less than m_1.

53. To replace a compound curve by a curve with spirals. The numerical illustration given below employs another method. We first solve for m_1 for the sharper branch of the curve, placing $\Delta_1 = \frac{1}{2}\Delta$ in Eq. 38. A value for R_2' may be found whose corresponding value of m_2 will equal m_1. Solving Eq. 38 for R', we obtain

$$R' = \frac{R \operatorname{vers} \frac{1}{2}\Delta - m \cos \frac{1}{2}\Delta - x}{\cos \phi - \cos \frac{1}{2}\Delta}. \quad . \quad . \quad . \quad (43)$$

Digitized by Google

Substituting in this equation the known value of m_1 ($=m_2$) and calling $R'=R_2'$, $R=R_2$, and $\Delta_2=\frac{1}{2}\Delta$, solve for R_2'. Obtain the value of AQ for each branch of the curve separately by Eq. 39, and compare the lengths of the old and new lines.

Example. Assume a compound curve with $D_1=8°$, $D_2=4°$, $\Delta_1=36°$, and $\Delta_2=32°$. Use 1°-per-25-feet spirals; $\phi_1=7°\ 0'$; $\phi_2=1°\ 30'$. Assume that the sharper curve is sharpened from 8° 0′ to 8° 12′.

[Eq. 38]

		R_1	2.85538
		exsec 36°	9.37303
169.209			2.22842
$R_1'=$699.326			
868.535		R_1'	2.84468
		cos ϕ_1	9.99675
		sec Δ_1	0.09204
	857.970		2.93347
		x_1	0.88241
		sec Δ_1	0.09204
	9.429		0.97445
867.399	867.399		
$m_1=$ 1.136			

[Eq. 43]

		R_2	3.15615
		vers 32°	9.18170
217.700			2.33785
		$m_1=1.136$	0.05538
		cos 32°	9.92842
	0.963		9.98380
	$x_2=$0.763		
1.726	1.726		
215.974			2.33440

		nat. cos ϕ = .99966	
		nat. cos $\Delta_2=$.84805	
		.15161	9.18073
$R_2'=1424.54$	[4° 1′ 22″]		3.15367
[Eq. 39] $v_1=174.722$		R_1'	2.84468
		sin ϕ_1	9.08580
	85.226		1.93057
		R_1'	2.84468
		cos ϕ_1	9.99675
		tan $\frac{1}{2}\Delta$ [$\Delta_1=36°$]	9.86126
504.302			2.70269
		$R_1=716.779$	
		$x_1=$ 7.628	
		709.151	2.85074
		tan $\frac{1}{2}\Delta$	9.86126
679.024			
600.461	515.235		2.71200
$AQ_1=$ 78.563	600.461		

Digitized by Google

[Eq. 39]

	R_2'	3.15367
$y_2 = 74.994$	$\sin \phi_2$	8.41792
37.290		1.57159
	R_2'	3.15367
	$\cos \phi_2$	9.99985
	$\tan \frac{1}{2}d\ (d_2 = 32°)$	9.79579
889.843		2.94931
	$R_2 = 1432.69$	
	$x_2 = 0.76$	
	1431.93	3.15592
	$\tan \frac{1}{2}d$	9.79579
894.770		2.95171
964.837		
932.060		
$AQ_2 = 32.777$		

For the length of the old track we have:

$$100\frac{d_1}{D_1} = 100\frac{36°}{8°} = 450.$$

$$100\frac{d_2}{D_2} = 100\frac{32°}{4°} = 800.$$

$AQ_1 =$	78.563
$AQ_2 =$	32.777
	1361.340

For the length of the new track we have:

$100\frac{d_1 - \phi_1}{D_1'}$	$= 100\frac{29°}{8°.20} =$	353.659
$100\frac{d_2 - \phi_2}{D_2'}$	$= 100\frac{30°.5}{4°.023} =$	758.140
Spiral on	8° 12′ curve	175.000
" "	4° 01′ 22″ "	75.
Length of new track		= 1361.799
" " old	"	= 1361.340
Excess in length of new track		= 0.459 feet.

Since the new track is slightly longer than the old, it shows that the new track runs too far *outside* the old track at the *P.C.C.* On the other hand the offset m is only 1.136. The maximum amount by which the new track comes *inside* of the old track at two points, presumably not far from Z' and Z, is very difficult to determine exactly. Since it is desirable that the maximum offsets (inside and outside) should be made as nearly equal as possible, this feature should not be sacrificed to an effort to make the two lines of precisely equal length so that the rails need not be cut. Therefore, if it is found that the offsets inside the old track are nearly equal to m (1.136), the above figures should stand. Otherwise m may be diminished (and the above excess in length of track diminished) by *increasing* R_1' very slightly and making the necessary consequent changes.

Digitized by Google

Use of Table IV. Prof. R. B. H. Begg, of Syracuse University, has submitted to the author a series of tables which will materially simplify the work of solving Equations (33) to (37), and which have been added to Table IV of previous editions. Since these equations involve R and Δ (which may each have *any* values) in combination with several values of ϕ, it would require impracticably extensive tables to give precise values of the required dimensions for *any* possible combination of R, Δ, and ϕ. But the tables may be utilized *by interpolation* with all necessary accuracy within their range. Rules for the use of the tables and for the field work are as follows:

1. Find *P. C.* (point A) as if no transition curve were to be used.
2. Lay off the distance AN (part C of table) to N; then offset the distance $A'N$ (part B) to A', the new *P. C.*; from N measure a distance NQ (part B) to Q.
3. Set transit on A'; sight parallel to tangent and run in circular curve, setting Z from deflection and distance (part B); or Z can be set by measuring ZK and QK (part B) from Q.
4. Set transit on Q, sight along tangent and turn the deflection (part A) for each 25-foot station, for as many chord lengths as required; or the points may be located by measuring distances y along the tangent from Q and offsetting the corresponding distances x

VERTICAL CURVES. / — 63

54. Necessity for their use. Whenever there is a change in the rate of grade, it is necessary to eliminate the angle that would be formed at the point of change and to connect the two grades by a curve. This is especially necessary at a sag between two grades, since the shock caused by abruptly forcing an upward motion to a rapidly moving heavy train is very severe both to the track and to the rolling stock. The necessity for vertical curves was even greater in the days when link couplers were in universal use and the "slack" in a long train was very great. Under such circumstances, when a train was moving down a heavy grade the cars would crowd ahead against the engine. Reaching the sag, the engine would begin to pull out, rapidly taking out the slack. Six inches of slack on each car would amount to several feet on a long train, and the resulting jerk on

Digitized by Google

the couplers, especially those near the rear of the train, frequently resulted in broken couplers or even derailments. A vertical curve will practically eliminate this danger if the curve is made long enough, but the rapidly increasing adoption of close spring couplers and air-brakes, even for freight trains, is obviating the necessity for such very long curves.

55. **Required length.** Theoretically the length should depend on the change in the rate of grade and on the length of the longest train on the road. A sharp change in the rate of grade requires a long curve; a long train requires a long curve; but since the longest trains are found on roads with light grades and small changes of grade, the required length is thus somewhat equalized. The A.R.E.A. rule is: "On class A roads (see § 198) rates of change of 0.1 per cent per station on summits and 0.05 per cent per station in sags should not be exceeded. On minor roads 0.2 per cent per station on summits and 0.1 per cent per station in sags may be used." When changing from a down grade to an up grade (or vice versa) the change of grade equals the numerical sum of the two rates of grade. For example, if a 0.5 per cent down grade is followed by a 0.7 per cent up grade, the road being a "minor" road, then, by the above rule the length of the curve should be at least $[0.5-(-0.7)]\div 0.1=12$ stations or 1200 feet. Added length increases the amount of earthwork required both in cuts and fills, but the resulting saving in operating expenses will always justify a considerable increase.

56. **Form of curve.** In Fig. 36 assume that A and C, equi-

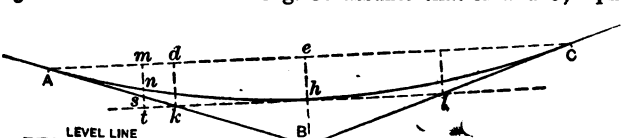


FIG. 36.

distant from B, are the extremities of the vertical curve. Bisect AC at e; draw Be and bisect it at h. Bisect AB and BC at k and l. The line kl will pass through h. A parabola may be drawn with its vertex at h which will be tangent to AB and BC at A and C. It may readily be shown * from the properties of a parabola that if an ordinate be drawn at *any* point (as at n) we will have

* See note at foot of p. 64.

Digitized by Google

$$sn : eh \text{ (or } hB) : : \overline{Am^2} : \overline{Ae}^2$$

or
$$sn = eh\frac{\overline{Am^2}}{\overline{Ae^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

In Fig. 36 the grades are necessarily exaggerated enormously. With the proportions found in practice we may assume that ordinates (such as *mt*, *eB*, etc.) are perpendicular to either grade, as may suit our convenience, without any appreciable error. In the numerical case given below, the variation of these ordinates from the vertical is 0° 07′, while the effect of this variation on the calculations in this case (as in the most extreme cases) is absolutely inappreciable. It may easily be shown that the angle CAB = half the *algebraic difference* of the rates of grade. Call the difference, expressed in per cent of grade, r; then $CAB = \frac{1}{2}r$. Let l = length (in "stations" of 100 feet) of the line AC, which is practically equal to the horizontal measurement. Since the angle CAB is one-half the total change of grade at B, it follows that $Be = \frac{1}{2}l \times \frac{1}{2}r$ Therefore

$$Bh = \tfrac{1}{8}lr. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44a)$$

Since Bh (or eh) are constant for any one curve, the correction sn at any point (see Eq. 44) equals a constant times Am^2.

57. Numerical example. Assume that B is located at Sta. 16+20; that the grade of AB is −0.5%, and of BC +0.7%; also that the elevation of B above the datum plane is 162.6. Then the algebraic difference of the grades, r, $= 0.7 - (-0.5) = 1.2$; $l = 12$. $Bh = \frac{1}{8}lr = \frac{1}{8} \times 12 \times 1.2 = 1.8$. A is at Sta. 10+20, and its elevation is $162.6 + (6 \times 0.5) = 165.6$; C is at Sta. 22+20 and its elevation is $162.6 + (6 \times 0.7) = 166.8$. The elevation of Sta. 11 is found by adding sn to the elevation of s on the straight grade line. The constant $(eh \div \overline{Ae}^2)$ equals in this case $1.8 \div 600^2 = \frac{1}{200000}$. Therefore the curve elevations are

A, Sta.	10+20,	$162.6 + (6.00 \times 0.5)$	$= 165.60$
	11	$165.6 - (0.80 \times 0.5) + \frac{1}{200000}\ 80^2$	$= 165.23$
	12	$165.6 - (1.80 \times 0.5) + \frac{1}{200000}\ 180^2$	$= 164.86$
	13	$165.6 - (2.80 \times 0.5) + \frac{1}{200000}\ 280^2$	$= 164.59$
	14	$165.6 - (3.80 \times 0.5) + \frac{1}{200000}\ 380^2$	$= 164.42$
	15	$165.6 - (4.80 \times 0.5) + \frac{1}{200000}\ 480^2$	$= 164.35$
	16	$165.6 - (5.80 \times 0.5) + \frac{1}{200000}\ 580^2$	$= 164.38$

Digitized by Google

B,	16+20, $162.6+1.80$	$=164.40$
	17 $166.8-(5.20\times0.7)+\frac{1}{200000}520^2$	$=164.51$
	18 $166.8-(4.20\times0.7)+\frac{1}{200000}420^2$	$=164.74$
	19 $166.8-(3.20\times0.7)+\frac{1}{200000}320^2$	$=165.07$
	20 $166.8-(2.20\times0.7)+\frac{1}{200000}220^2$	$=165.50$
	21 $166.8-(1.20\times0.7)+\frac{1}{200000}120^2$	$=166.03$
	22 $166.8-(0.20\times0.7)+\frac{1}{200000}20^2$	$=166.66$
C,	22+20, $162.6+(6.00\times0.7)$	$=166.80$

DEMONSTRATION OF EQ. 44.

The general equation of a parabola passing through the point n (Fig. 36) may be written

$$y^2 + y_n^2 = 2p(x + x_n),$$

from which

$$x_n = \frac{y^2}{2p} + \frac{y_n^2}{2p} - x.$$

When $x = x_A$, $y = y_A$, and we have

$$x_n = \frac{y_A^2}{2p} + \frac{y_n^2}{2p} - x_A.$$

The general equation of a tangent passing through the point A may be written

$$yy_A = p(x + x_A),$$

from which

$$x = \frac{yy_A}{p} - x_A.$$

When $x = x_s$, $y = y_s[= y_n]$, and we have

$$x_s = \frac{y_n y_A}{p} - x_A.$$

$$\overline{sn} = x_n - x_s = \frac{y_A^2 + y_n^2 - 2y_n y_A}{2p}$$

$$= \frac{(y_A - y_n)^2}{2p} = \frac{\overline{Am}^2}{2p},$$

$$2p = \frac{y_A^2}{x_A} = \frac{\overline{Ae}^2}{\overline{eh}}.$$

$$\therefore \quad \overline{sn} = \frac{\overline{Am}^2}{\overline{Ae}^2}\overline{eh},$$

This proves the general proposition that if secants are drawn parallel to the axis of x, intersecting a parabola and a tangent to it, the intercepts between the tangent and the parabola are proportional to the square of the distances (measured parallel to y) from the tangent point.

Digitized by Google

CHAPTER III.

EARTHWORK. — 71

FORM OF EXCAVATIONS AND EMBANKMENTS.

58. Usual form of cross-section in cut or fill. The normal form of cross-section in cut is as shown in Fig. 37, in which $e \ldots g$ represents the natural surface of the ground, no matter

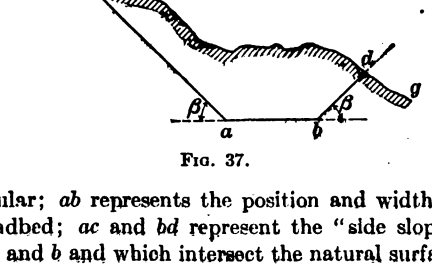


FIG. 37.

how irregular; ab represents the position and width of the required roadbed; ac and bd represent the "side slopes" which begin at a and b and which intersect the natural surface at such

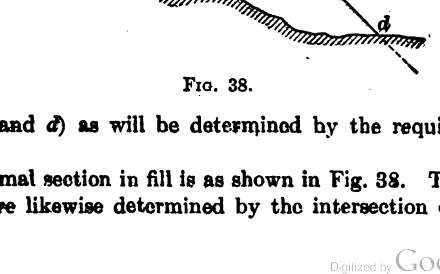


FIG. 38.

points (c and d) as will be determined by the required slope angle (β).

The normal section in fill is as shown in Fig. 38. The points c and d are likewise determined by the intersection of the ro-

Digitized by Google

quired side slopes with the natural surface. In case the required roadbed (*ab* in Fig. 39) intersects the natural surface, both cut

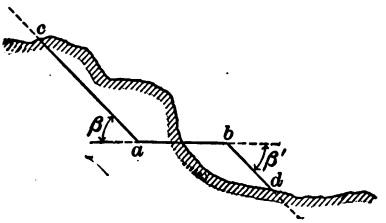


FIG. 39.

and fill are required, and the points *c* and *d* are determined as before. Note that β and β' are not necessarily equal. Their proper values will be discussed later.

59. **Terminal pyramids and wedges.** Fig. 40 illustrates the general form of cross-sections when there is a transition from cut to fill. *a* . . . *g* represents the grade line of the road which

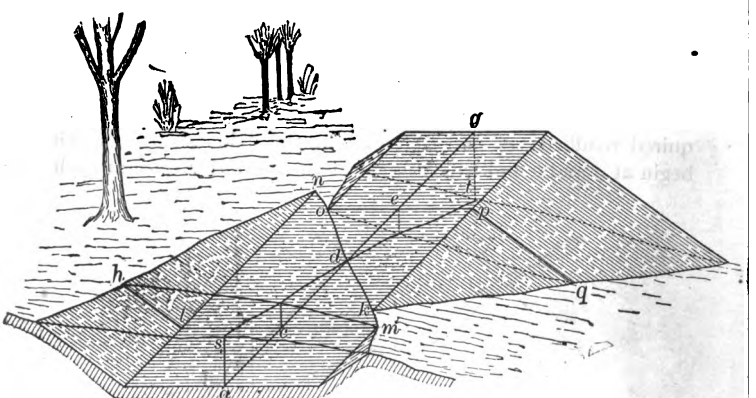


FIG. 40.

passes from cut to fill at *d*. *sdt* represents the surface profile. A cross-section taken at the point where either side of the roadbed *first* cuts the surface (the point *m* in this case) will usually be triangular if the ground is regular. A similar cross-section should be taken at *o*, where the other side of the roadbed cuts the surface. In general the earthwork of cut and fill terminates

Digitized by Google

in two pyramids. In Fig. 40 the pyramid vertices are at *n* and *k*, and the bases are *lhm* and *opq*. The roadbed is generally wider in cut than in fill, and therefore the section *lhm* and the altitude *ln* are generally greater than the section *opq* and the altitude *pk*. When the line of intersection of the roadbed and natural surface (*nodkm*) becomes perpendicular to the axis of the roadbed (*ag*) the pyramids become wedges whose bases are the nearest convenient cross-sections.

60. **Slopes.** **a. Cuttings.** The required slopes for cuttings vary from perpendicular cuts, which may be used in hard rock which will not disintegrate by exposure, to a slope of perhaps 4 horizontal to 1 vertical in a soft material like quicksand or in a clayey soil which flows easily when saturated. For earthy materials a slope of 1 : 1 is the maximum allowable, and even this should only be used for firm material not easily affected by saturation. A slope of 1½ horizontal to 1 vertical is a safer slope for average earthwork. It is a frequent blunder that slopes in cuts are made too steep, and it results in excessive work in clearing out from the ditches the material that slides down, at a much higher cost per yard than it would have cost to take it out at first, to say nothing of the danger of accidents from possible landslides.

b. Embankments. The slopes of an embankment vary from 1 : 1 to 1.5 : 1. A rock fill will stand at 1 : 1, and if some care is taken to form the larger pieces on the outside into a rough dry wall, a much steeper slope can be allowed. This method is sometimes a necessity in steep side-hill work. Earthwork embankments generally require a slope of 1½ to 1. If made steeper at first, it generally results in the edges giving way, requiring repairs until the ultimate slope is nearly or quite 1½ : 1. The difficulty of incorporating the added material with the old embankment and preventing its sliding off frequently makes these repairs disproportionately costly.

61. **Compound sections.** When the cut consists partly of earth and partly of rock, a compound cross-section must be made. If borings have been made so that the contour of the rock surface is accurately known, then the true cross-section may be determined. The rock and earth should be calculated separately, and this will require an accurate knowledge of where the rock "runs out"—a difficult matter when it must be deter-

Digitized by Google

mined by boring. During construction the center part of the earth cut would be taken out first and the cut widened until a sufficient width of rock surface had been exposed so that the rock cut would have its proper width and side slopes. Then the earth slopes could be cut down at the proper angle. A "berm" of about three feet is usually left on the edges of the rock cut as

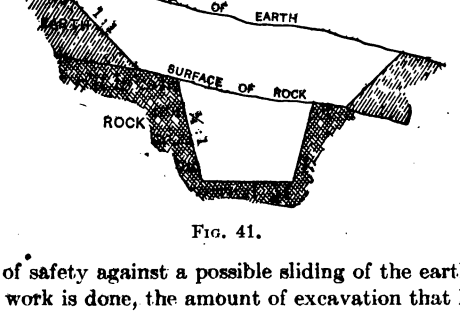


FIG. 41.

a margin of safety against a possible sliding of the earth slopes. After the work is done, the amount of excavation that has been made is readily computable, but accurate preliminary estimates are difficult. The area of the cross-section of earth in the figure must be determined by a method similar to that developed for borrow-pits (see § 89).

62. Width of roadbed. Owing to the large and often disproportionate addition to volume of cut or fill caused by the addition of even one foot to the width of roadbed, there is a natural tendency to reduce the width until embankments become unsafe and cuts are too narrow for proper drainage. The cost of maintenance of roadbed is so largely dependent on the drainage of the roadbed that there is true economy in making an ample allowance for it. The practice of some of the leading railroads of the country in this respect is given in the following table, in which are also given some data belonging more properly to the subject of superstructure.

It may be noted from the table that the average width for an *earthwork* cut, single track, is about 24.7 feet, with a minimum of 19 feet 2 inches. The widths of fills, single track, average over 18 feet, with numerous minimums of 16 feet. The widths for double track may be found by adding the distance between track centers, which is usually 13 feet.

Digitized by Google

WIDTH OF ROADBED FOR SINGLE AND DOUBLE TRACK—SLOPE RATIOS—DISTANCES BETWEEN TRACK CENTERS.

Road.	Single Track.		Double Track.		Slope Ratios.		Distance Between Track Centers.
	Cut.	Fill.	Cut.	Fill.	Cut.	Fill.	
A., T. & Santa Fé	{ 28′ earth { 22′ rock	20			1 : 1 ¼ : 1	1.5 : 1	
Chicago, Burlington & Quincy	14+(2×5)*	16	28+(2×5)	30	1.5 : 1	1.5 : 1	14′
Chicago, Milwaukee & St. Paul	18+(2×6)	20 to 24	31+(2×6)	33 to 37	1.5 : 1	1.5 : 1	13′
C., C., C. & St. Louis	20+(2×4)	20	33+(2×4)	33	1.5 : 1	1.5 : 1	13′
Illinois Central	32.5	18			1.5 : 1	1.5 : 1	
Erie	20′ 8½″	20′ 8½″	33′ 8½″	33′ 8½″	1.5 : 1	1.5 : 1	13′
Lehigh Valley	14+(2×3.5)	16	27+(2×3.5)	30	1 : 1	1.5 : 1	13′
Lake Shore & Michigan Southern			33+(2×7.25)	32	1.5 : 1	1.5 : 1	13′
Louisville & Nashville	13+(2×4.5)	16			1 : 1	1.5 : 1	
Michigan Central			(33+2×2.5) 30	33 30	1.5 : 1 1.5 : 1	1.5 : 1 1.5 : 1	13′ 12′
N. Y., N. H. & H.							
Norfolk & Western	{ 21′ 2″ earth { 16′ rock	17′ 2″	34′ 2″ earth 29′ rock	30′ 2′	1.5 : 1 ¼ : 1 ½ : 1	1.5 : 1	13′ 13′
Pennsylvania	{ 19′ 2″ light traffic { 27′ 2″ heavy "	19′ 2″ 19′ 2″	31′ 4″+(2×4)	31′ 4″	1.5 : 1	1.5 : 1	12′ 2″
Union Pacific	14+(2×3.5)	16			1 : 1	1.5 : 1	

* (2×5) signifies two ditches each 5 feet wide; the following cases should be interpreted similarly.

Digitized by Google

63. Form of subgrade. Specifications (or the cross-section drawings) formerly required that the subgrade should have a curved form, convex upward, or that it should slope outward from a slight ridge in the center, with the evident purpose of draining to the sides all water which might percolate through the ballast. If the subsoil were hard and impenetrable by the ballast, the method might answer, but experience has shown that, with ordinary subsoils, the ballast immediately under each rail is forced a little deeper into the subsoil by the passage of each train. Periodical retamping of ballast under the ends of the ties, and little or no tamping under the center, only adds to the accumulation under each rail. A cross-section of a very old roadbed will frequently show twice as much depth of ballast under the rails as there is under the center. This method of tamping quickly obliterates the original line of demarcation between ballast and subsoil and any expected improvement in drainage due to sloping subsoil is not realized. Therefore the A.R.E.A. specifications call for *flat* subgrades.

64. Ditches. "The stability of the track depends upon the strength and permanence of the roadbed and structures upon which it rests; whatever will protect them from damage or prevent premature decay should be carefully observed. The worst enemy is WATER, and the further it can be kept away from the track, or the sooner it can be diverted from it, the better the track will be protected. Cold is damaging only by reason of the water which it freezes; therefore the first and most important provision for good track is drainage." (Rules of the Road Department, Illinois Central R. R.)

The form of ditch generally prescribed has a flat bottom 12″ to 24″ wide and with sides having a minimum slope, except in rock-work, of 1 : 1, more generally 1.5 : 1 and sometimes 2 : 1. Sometimes the ditches are made V-shaped, which is objectionable unless the slopes are low The best form is evidently that which will cause the greatest flow for a given slope, and this will evidently be the form in which the ratio of area to wetted perimeter is the largest. The semicircle fulfills this condition better than any other form, but the nearly vertical sides would be difficult to maintain. (See Fig. 42.) A ditch, with a flat bottom and such

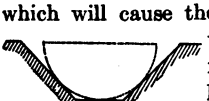


FIG. 42.

Digitized by Google

slopes as the soil requires, which approximates to the circular form will therefore be the best.

When the flow will probably be large and at times rapid it will be advisable to pave the ditches with stone, especially if the soil is easily washed away. Six-inch tile drains, placed 2′ under the ditches, are prescribed on some roads. (See Fig. 43.) No better method could be devised to insure a dry subsoil. The ditches through cuts should be led off at the end of the cut so that the adjacent embankment will not be injured.

Wherever there is danger that the drainage from the land above a cut will drain down into the cut, a ditch should be made near the edge of the cut to intercept this drainage, and this ditch should be continued, and paved if necessary, to a point where the outflow will be harmless. Neglect of these simple and inexpensive precautions frequently causes the soil to be loosened on the shoulders of the slopes during the progress of a heavy rain, and results in a landslide which will cost more to repair than the ditches which would have prevented it for all time.

Ditches should be formed along the bases of embankments; they facilitate the drainage of water from the embankment, and may prevent a costly slip and disintegration of the embankment.

65. Effect of sodding the slopes, etc. Engineers are unanimously in favor of rounding off the shoulders and toes of embankments and slopes, sodding the slopes, paving the ditches, and providing tile drains for subsurface drainage, all to be put in during original construction. (See Fig. 43.) Some of the highest grade specifications call for the removal of the top layer of vegetable soil from cuts and from under proposed fills to some convenient place, from which it may be afterwards spread on the slopes, thus facilitating the formation of sod from grass-seed. But while engineers favor these measures and their economic value may be readily demonstrated, it is generally impossible to obtain the authorization of such specifications from railroad directors and promoters. The addition to the original cost of the roadbed is considerable, but is by no means as great as the capitalized value of the extra cost of maintenance resulting from the usual practice. Fig. 43 is a copy of

Digitized by Google

designs * presented at a convention of the American Society of Civil Engineers by Mr. D. J. Whittemore, Past President of the Society and Chief Engineer of the Chi., Mil. & St. Paul

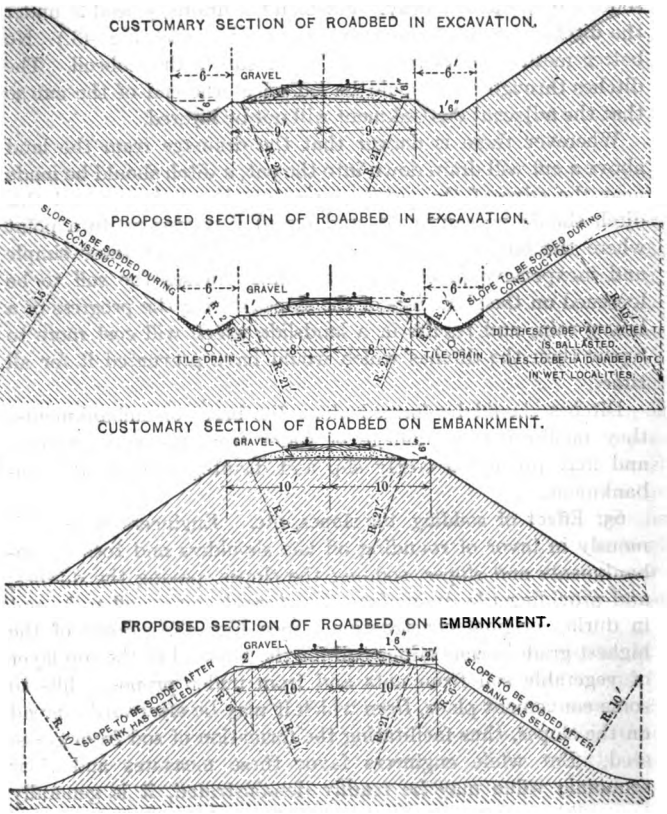

CUSTOMARY SECTION OF ROADBED IN EXCAVATION.
GRAVEL
6'
9'
PROPOSED SECTION OF ROADBED IN EXCAVATION.
SLOPE TO BE SODDED DURING CONSTRUCTION.
TILE DRAIN
DITCHES TO BE PAVED WHEN TI IS BALLASTED.
TILES TO BE LAID UNDER DITC IN WET LOCALITIES.
CUSTOMARY SECTION OF ROADBED ON EMBANKMENT.
PROPOSED SECTION OF ROADBED ON EMBANKMENT.
SLOPE TO BE SODDED AFTER BANK HAS SETTLED.

FIG. 43.—"WHITTEMORE ON RAILWAY EXCAVATION AND EMBANKMENTS" Trans. Am. Soc. C. E., Sept. 1894.

R. R. The "customary sections" represent what is, with some variations of detail, the practice of many railroads. The "pro-

* Trans. Am. Soc. Civil Eng., Sept. 1894.

Digitized by Google

posed sections" elicited unanimous approval. They should be adopted when not prohibited by financial considerations.

EARTHWORK SURVEYS.

66. Relation of actual volume to the numerical result. It should be realized at the outset that the accuracy of the result of computations of the volume of any given mass of earthwork has but little relation to the accuracy of the mere numerical work. The process of obtaining the volume consists of two distinct parts. In the first place it is assumed that the volume of the earthwork may be represented by a more or less complicated geometrical form, and then, secondly, the volume of such a geometrical form is computed. A desire for simplicity (or a frank willingness to accept approximate results) will often cause the cross-section men to assume that the volume may be represented by a very simple geometrical form which is really only a very rough approximation to the true volume. In such a case, it is only a waste of time to compute the volume with minute numerical accuracy. One of the first lessons to be learned is that economy of time and effort requires that the accuracy of the numerical work should be kept proportional to the accuracy of the cross-sectioning work, and also that the accuracy of both should be proportional to the use to be made of the results. The subject is discussed further in § 94.

67. Prismoids. To compute the volume of earthwork, it is necessary to assume that it has some geometric form whose volume is readily determinable. The general method is to consider the volume as consisting of a series of *prismoids*, which are solids having parallel plane ends and bounded by surfaces which may be formed by lines moving continuously along the edges of the bases These surfaces may also be considered as the surfaces generated by lines moving along the edges joining the corresponding points of the bases, these edges being the directrices, and the lines being always parallel to either base, which is a plane director. The surfaces thus developed may or may not be planes. The volume of such a prismoid is readily determinable (as explained in § 70 *et seq.*), while its definition is so very general that it may be applied to very rough ground. The "two plane ends" are sections perpendicular to the axis of the road. The roadbed and side slopes (also plane) form three of

Digitized by Google

the side surfaces. The only approximation lies in the degree of accuracy with which the plane (or warped) surfaces coincide with the actual surface of the ground between these two sections. This accuracy will depend (*a*) on the number of points which are taken in each cross-section and the accuracy with which the lines joining these points coincide with the actual cross-sections; (*b*) on the skill shown in selecting places for the cross-sections so that the warped surfaces shall coincide as nearly as possible with the surface of the ground. In fairly smooth country, cross-sections every 100 feet, placed at the even stations, are sufficiently accurate, and such a method simplifies the computations greatly; but in rough country cross-sections must be interpolated as the surface demands. As will be explained later, carelessness or lack of judgment in cross-sectioning will introduce errors of such magnitude that all refinements in the computations are utterly wasted.

68. Cross-sectioning. The process of cross-sectioning consists in determining at any place the intersection by a vertical plane of the prism of earth lying between the roadbed, the side slopes, and the natural surface. The intersection with the road-

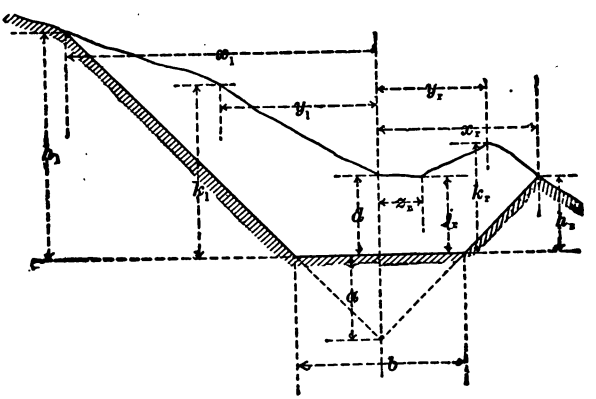


FIG. 44.

bed and side slopes gives three straight lines. The intersection with the natural surface is in general an irregular line. On smooth regular ground or when approximate results are acceptable this line is assumed to be straight. According to the irreg-

Digitized by Google

ularity of the ground and the accuracy desired more and more "intermediate points" are taken.

The distance (d in Fig. 44) of the roadbed below (or above) the natural surface at the center is known or determined from the profile or by the computed establishment of the grade line. The distances out from the center of all "breaks" are determined with a tape. To determine the elevations for a cut, set up a level at any convenient point so that the line of sight is higher than any point of the cross-section, and take a rod reading on the center point. This rod reading added to d gives the height of the instrument (H. I.) above the roadbed. Subtracting from H. I. the rod reading at any "break" gives the height of that point above the roadbed (h_l, k_l, h_r, etc.). This is true for all cases in excavation. For fill, the rod reading at center minus d equals the H. I., which may be positive or negative. When negative, add to the "H. I." the rod readings of the intermediate points to get their depths below "grade"; when positive, subtract the "H. I." from the rod readings.

The heights or depths of these intermediate points above or below grade need only be taken to the nearest tenth of a foot, and the distances out from the center will frequently be sufficiently exact when taken to the nearest foot. The roughness of the surface of farming land or woodland generally renders useless any attempt to compute the volume with any greater accuracy than these figures would imply unless the form of the ridges and hollows is especially well defined. The position of the slope-stake points is considered in the next section. Additional discussion regarding cross-sectioning is found in § 82.

69. Position of slope-stakes. The slope-stakes are set at the intersection of the required side slopes with the natural surface, which depends on the center cut or fill (d). The distance of the slope-stake from the center for the lower side is $x = \frac{1}{2}b + s(d+y)$; for the up-hill side it is $x' = \frac{1}{2}b + s(d-y')$. s is the "slope ratio" for the side slopes, the ratio of horizontal to vertical. In the above equation both x and y are unknown. Therefore some position must be found by trial which will satisfy the equation. As a preliminary, the value of x for the point $a = \frac{1}{2}b + sd$, which is the value of x for *level* cross-sections. In the case of fills on sloping ground the value of x on the *down-hill* side is *greater* than this; on the *up-hill* side it is *less*. The difference in distance is s times the difference of elevation. Take a

Digitized by Google

numerical case corresponding with Fig. 45. The rod reading on c is 2.9; $d=4.2$; therefore the telescope is $4.2-2.9=1.3$ *below* grade. $s=1.5:1$, $b=16$. Hence for the point a (or for level ground) $x=\frac{1}{2}\times 16+1.5\times 4.2=14.3$. At a distance out of 14.3 the ground is seen to be about 3 feet lower, which will not only require $1.5\times 3=4.5$ more, but enough additional distance so that the added distance shall be 1.5 times the additional drop. As a first trial the rod may be held at 24 feet out and a reading of, say, 8.3 is obtained. $8.3+1.3=9.6$, the depth of the point below grade. The point on the slope line (n) which has this depth below grade is at a distance from the center

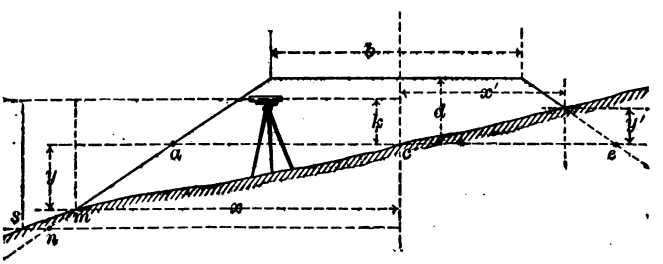


FIG. 45.

$x=8+1.5\times 9.6=22.4$. The point on the surface (s) having that depth is 24 feet out. Therefore the true point (m) is nearer the center. A second trial at 20.5 feet out gives a rod reading of, say, 7.1 or a depth of 8.4 below grade. This corresponds to a distance out of 20.6. Since the natural soil (especially in farming lands or woods) is generally so rough that a difference of elevation of a tenth or so may be readily found by slightly varying the location of the rod (even though the distance from the center is the same), it is useless to attempt too much refinement, and so in a case like the above the combination of 8.4 below grade and 20.6 out from center may be taken to indicate the proper position of the slope-stake. This is usually indicated in the form of a fraction, the distance out being the denominator and the height above (or below) grade being the numerator; the fact of *cut* or *fill* may be indicated by C or F. Ordinarily a second trial will be sufficient to determine with sufficient accuracy the true position of the slope-stake. Experienced men will frequently estimate the required distance

Digitized by Google

out to within a few tenths at the first trial. The left-hand pages of the note-book should have the station number, surface elevation, grade elevation, center cut or fill, and rate of grade. The right-hand pages should be divided in the center and show the distances out and heights above grade of all points, as is illustrated in § 84. The notes should read UP the page, so that when looking ahead along the line the figures are in their proper relative position. The "fractions" farthest from the center line represent the slope-stake points.

70. Setting slope-stakes by means of "automatic" slope-stake rods. The equipment consists of a specially graduated tape and a specially constructed rod. The tape may readily be prepared by marking on the *back* side of an ordinary 50-foot tape which is graduated to feet and tenths. Mark "0" at "$\frac{1}{2}b$" from the tape-ring. Then graduate from the zero backward, at true scale, to the ring. Mark off "feet" and "tenths" on a scale proportionate to the slope ratio. For example, with the usual slope ratio of 1.5:1 each "foot" would measure 18 inches and each "tenth" in proportion.

The rod, 10 feet long, is shod at each end and has an endless tape passing within the shoes at each end and over pulleys—to reduce friction. The tape should be graduated in feet and tenths, from 0 to 20 feet—the 0 and 20 coinciding. By moving the tape so that 0 is at the bottom of the rod—or (practically) so that the 1-foot mark on the tape is one foot above the bottom of the shoe, an index mark may be placed on the back of the rod (say at 15—on the tape) and this readily indicates when the tape is "set at zero."

The method of use may best be explained from the figure and from the explicit rules as stated. The proof is given for two assumed positions of the level.

(1) Set up the level so that it is higher than the "center" and (if possible) higher than both slope-stakes, but not more than a rod-length higher. On very steep ground this may be impossible and each slope-stake must be set by separate positions of the level.

(2) Set the rod-tape at zero (i.e., so that the 15-foot mark on the *back* is at the index mark).

(3) Hold the rod at the center-stake (B) and note the reading (n_1 or n_2). Consider n to be always plus; consider d to be plus for cut and minus for fill.

Digitized by Google

(4) *Raise* the tape on the *face* side of the rod $(n+d)$. Applied literally (and algebraically), when the level is *below* the roadbed (only possible for fill), $(n+d)=(n_2+(-d_f))=n_2-d_f$. This being numerically negative, the tape is *lowered* (d_f-n_2). With level at (1), for fill, $(n+d)=(n_1+(-d_f))=(n_1-d_f)$; this being positive, the tape is raised. With level at (1), for cut, the tape is raised (n_1+d_c). In every case the effect is the same as if the telescope were set at the elevation of the roadbed.

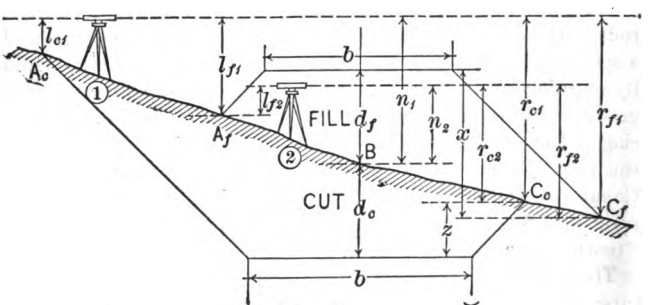


FIG. 45*a*.

(5) With the special distance-tape, so held that its zero is $\frac{1}{2}b$ from the center, carry the rod out until the rod reading equals the reading indicated by the tape. Since in cut the tape is raised $(n+d)$, the zero of the rod-tape is always higher than the level (unless the rod is held at or below the elevation of the roadbed—which is only possible on side-hill work), and the reading at either slope-stake is necessarily *negative*. The reading for slope-stakes in fill is always positive.

(6) Record the rod-tape reading as the numerator of a fraction and the *actual distance* out (read directly from the *other* side of the distance-tape) as the denominator of the fraction.

Proof. Fill. Level at (1). Tape is raised (n_1-d_f). When rod is held at C_f, the rod reading is $+x$, which $=r_{f1}-(n_1-d_f)$. But the reading on the back side of the distance-tape is also x.

Fill. Level at (2). Tape is raised (n_2-d_f), i.e., it is *lowered* (d_f-n_2). When rod is held at C_f, the rod reading is $+x$, which similarly $= r_{f2}-(n_2-d_f) = r_{f2}+(d_f-n_2)$. Distance-tape as before.

Digitized by Google

Cut Level at (1). Tape is raised (n_1+d_c). When rod is held at C_c the rod reading is $-z$, which $= r_{c_1}-(n_1+d_c)$, i.e., $z=(n_1+d_c)-r_{c_1}$. The distance-tape will read z.

Side-hill work. It is easily demonstrated that the method, when followed literally, may be applied to side-hill work, although there is considerable chance for confusion and error, when, as is usual, $\frac{1}{2}b$ and the slope ratio are different for cut and for fill.

The method appears complicated at first, but it becomes mechanical and a time-saver when thoroughly learned. The advantages are especially great when the ground is fairly level transversely, but decrease when the difference of elevation of the center and the slope-stake is more than the rod length. By setting the rod-tape "at zero," the rod may always be used as an ordinary level rod and the regular method adopted, as in § 69. Many engineers who have thoroughly tested these rods are enthusiastic in their praise as a time-saver.

COMPUTATION OF VOLUME.

71. Prismoidal formula. Let Fig. 46 represent a triangular prismoid. The two triangles forming the ends lie in *parallel* planes, but since the angles of one triangle are not equal to the corresponding angles of the other triangle, at least two of the surfaces must be *warped*. If a section, parallel to the bases, is

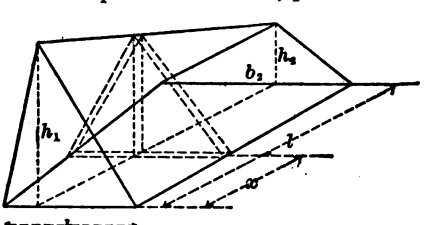


FIG. 46.

made at any point at a distance x from one end, the area of the section will evidently be

$$A_x=\tfrac{1}{2}b_xh_x=\tfrac{1}{2}\left[b_1+(b_2-b_1)\frac{x}{l}\right]\left[h_1+(h_2-h_1)\frac{x}{l}\right]$$

Digitized by Google

The volume of a section of infinitesimal length will be $A_x dx$, and the total volume of the prismoid will be *

$$\int_0 A_x dx = \tfrac{1}{2}\int_0^l \left[b_1+(b_2-b_1)\frac{x}{l}\right]\left[h_1+(h_2-h_1)\frac{x}{l}\right] dx$$

$$= \tfrac{1}{2}\left[b_1h_1x+(b_2-b_1)h_1\frac{x^2}{2l}+b_1(h_2-h_1)\frac{x^2}{2l} + (b_2-b_1)(h_2-h_1)\frac{x^3}{3l^2}\right]_0^l$$

$$= \tfrac{1}{2}\left\{b_1h_1l+[(b_2-b_1)h_1+b_1(h_2-h_1)]\frac{l}{2}+(b_2-b_1)(h_2-h_1)\frac{l}{3}\right\}$$

$$= \frac{l}{2}[\tfrac{1}{3}b_1h_1+\tfrac{1}{6}b_1h_2+\tfrac{1}{6}b_2h_1+\tfrac{1}{3}b_2h_2]$$

$$= \frac{l}{6}[\tfrac{1}{2}b_1h_1+\tfrac{1}{2}b_1(h_1+h_2)+\tfrac{1}{2}b_2(h_1+h_2)+\tfrac{1}{2}b_2h_2]$$

$$= \frac{l}{6}\left[\tfrac{1}{2}b_1h_1+4\left(\frac{1}{2}\cdot\frac{b_1+b_2}{2}\cdot\frac{h_1+h_2}{2}\right)+\tfrac{1}{2}b_2h_2\right]$$

$$= \frac{l}{6}[A_1+4A_m+A_2], \quad \ldots\ldots\ldots\ldots \quad (45)$$

in which A_1, A_2, and A_m are the areas respectively of the two bases and of the middle section. Note that A_m is *not* the *mean* of A_1 and A_2, although it does not necessarily differ very greatly from it.

The above proof is absolutely independent of the values, absolute or relative, of b_1, b_2, h_1, or h_2. For example, h_2 may be zero and the second base reduces to a line and the prismoid becomes wedge-shaped; or b_2 and h_2 may both vanish, the second base becoming a point and the prismoid reduces to a pyramid. Since every prismoid (as defined in § 67) may be reduced to a combination of triangular prismoids, wedges, and pyramids, and

* Students unfamiliar with the Integral Calculus may take for granted the fundamental formulæ that $\int dx = x$, that $\int x\,dx = \frac{1}{2}x^2$, and that $\int x^2 dx = \frac{1}{3}x^3$; also that in integrating between the limits of l and 0 (zero), the value of the integral may be found by simply substituting l for x after integration.

Digitized by Google

since the formula is true for any one of them individually, it is true for all collectively; therefore it may be stated that *

The volume of a prismoid equals one sixth of the perpendicular distance between the bases multiplied by the sum of the areas of the two bases plus four times the area of the middle section.

While it is always possible to compute the volume of any prismoid by the above method, it becomes an extremely complicated and tedious operation to compute the true value of the middle section if the end sections are complicated in form. It therefore becomes a simpler operation to compute volumes by approximate formulæ and apply, if necessary, a correction. The most common methods are as follows:

72. Averaging end areas. The volume of the triangular prismoid (Fig. 46), computed by averaging end areas, is $\frac{l}{2}[\frac{1}{2}b_1h_1+\frac{1}{2}b_2h_2]$. Subtracting this from the true volume (as given in the equation above Eq. 45), we obtain the correction

$$\frac{l}{12}[(b_1-b_2)(h_2-h_1)]. \quad . \quad . \quad . \quad . \quad . \quad (46)$$

This shows that if either the h's or b's are equal, the correction vanishes; it also shows that if the bases are roughly similar and b varies roughly with h (which *usually* occurs, as will be seen later), the correction will be *negative*, which means that the method of averaging end areas *usually* gives *too large* results.

73. Middle areas. Sometimes the middle area is computed and the volume is assumed to be equal to the length times the middle area. This will equal $\frac{l}{2}\times\frac{b_1+b_2}{2}\times\frac{h_1+h_2}{2}$. Subtracting this from the true volume, we obtain the correction

$$\frac{l}{24}(b_1-b_2)(h_1-h_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

As before, the form of the correction shows that if either the h's or b's are equal, the correction vanishes; also under the *usual* conditions, as before, the correction is *positive* and only one-half as large as by averaging end areas. Ordinarily the labor involved in the above method is no less than that of applying the exact prismoidal formula.

* The student should note that the derivation of equation (45) does not complete the proof, but that the statements in the following paragraph are logically necessary for a general proof.

Digitized by Google

74. Two-level ground. When *approximate* computations of earthwork are sufficiently exact the field-work may be materially reduced by observing simply the center cut (or fill) and the natural slope a, measured with a clinometer. The area of such a section (see Fig. 48) equals

$$\tfrac{1}{2}(a+d)(x_l+x_r)-\frac{ab}{2}.$$

But

$$x_l \tan\beta = a+d+x_l \tan a,$$

from which

$$x_l = \frac{a+d}{\tan\beta-\tan a}.$$

Similarly,

$$x_r = \frac{a+d}{\tan\beta+\tan a}.$$

Substituting,

$$\text{Area} = (a+d)^2 \frac{\tan\beta}{\tan^2\beta-\tan^2 a} - \frac{ab}{2}. \quad . \quad . \quad . \quad (48)$$

The values a, $\tan\beta$, $\tan^2\beta$ are constant for all sections, so that it requires but little work to find the area of any section.

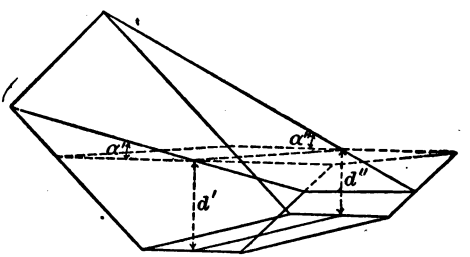


FIG. 47.

As this method of cross-sectioning implies considerable approximation, it is generally a useless refinement to attempt to com-

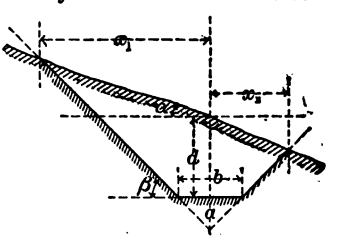


FIG. 48.

Digitized by Google

pute the volume with any greater accuracy than that obtained by averaging end areas. It may be noted that it may be easily proved that the correction to be applied is of the same form as that found in § 72 and equals

$$\frac{l}{12}[(x_l' + x_r') - (x_l'' + x_r'')]\,[(d'' + a) - (d' + a)],$$

which reduces to

$$\text{Corr.} = \frac{l}{6}\left\{\left[(a+d')\frac{\tan\beta}{\tan^2\beta - \tan^2\alpha'} - (a+d'')\frac{\tan\beta}{\tan^2\beta - \tan^2\alpha''}\right][d'' - d']\right\}. \quad (49)$$

When $d'' = d'$ the correction vanishes. This shows that when the center heights are equal there is no correction—regardless of the slope. If the slope is uniform throughout, the form of the correction is simplified and is invariably *negative*. Under the usual conditions the correction is *negative*, i.e., the method *generally* gives *too large* results.

75. Level sections. When the country is very level or when only approximate preliminary results are required, it is sometimes assumed that the cross-sections are level. The method of level sections is capable of easy and rapid computation. The area may be written as

$$(a+d)^2 s - \frac{ab}{2} \quad \ldots\ldots\ldots \quad (50)$$

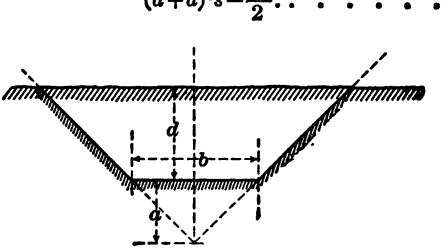


FIG. 49.

This also follows from Eq. 48 when $a = 0$ and $\tan\beta = \frac{1}{s}$. s here represents the "slope ratio," i.e., the ratio of the horizontal projection of the slope to the vertical. A table is very readily formed giving the area in square feet of a section of given depth and for any given width of roadbed and ratio of side-slopes.

Digitized by Google

The area may also be readily determined (as illustrated in the following example) without the use of such a table; a table of squares will facilitate the work. Assuming the cross-sections at equal distances ($=l$) apart, the total approximate volume for any distance will be

$$\frac{l}{2}[A_0+2(A_1+A_2+\ldots A_{n-1})+A_n]. \quad . \quad . \quad . \quad (51)$$

The prismoidal correction may be directly derived from Eq. 46 as $\frac{l}{12}[2(a+d')s-2(a+d'')s][(a+d'')-(a+d')]$, which reduces to

$$-\frac{ls}{6}(d'-d'')^2 \quad \text{or} \quad -\frac{1}{12}\frac{b}{a}(d'-d'')^2. \quad . \quad . \quad (52)$$

This may also be derived from Eq. 49, since $a=0$, $\tan a=0$, and $\tan\beta=2a\div b$ This correction is *always* negative, showing that the method of averaging end areas, when the sections are level, always gives too large results The prismoidal correction for any one prismoid is therefore a constant times the *square* of a difference. The squares are always positive whether the differences are positive or negative. The correction therefore becomes

$$-\frac{l}{12}\frac{b}{a}\Sigma(d'\sim d'')^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

76. Numerical example: level sections. Given the following center heights for the same number of consecutive stations 100 feet apart; width of roadbed 18 feet; slope 1½ to 1.

The products in the fifth column may be obtained very readily and with sufficient accuracy by the use of the slide-rule described in § 79. The products should be considered as $(a+d)(a+d)\div\frac{1}{s}$. In this problem $s=1\frac{1}{2}$, $\frac{1}{s}=.6667$. To apply the rule to the first case above, place 6667 on scale B over 89 on scale A, then opposite 89 on scale B will be found 118.8 on scale A. The position of the decimal point will be evident from an approximate mental solution of the problem.

Digitized by Google

Sta.	Center Height.	$a+d$	$(a+d)^2$	$(a+d)^2 s$	Areas.	$d' \sim d''$	$(d' \sim d'')^2$
17	2.9	8.9	79.21	118.81	118.81		
18	4.7	10.7	114.49	171.74	343.48	1.8	3.24
19	6.8	12.8	163.84	245.76	491.52	2.1	4.41
20	11.7	17.7	313.29	469.93	939.86	4.9	24.01
21	4.2	10.2	104.04	156.06	312.12	7.5	56.25
22	1.6	7.6	57.76	86.64	86.64	2.6	6.76
					$\times 2 =$ 2292.43		94.67

$$\frac{ab}{2} = \frac{6 \times 18}{2} = 54 \qquad 10 \times 54 = \frac{540}{1752.43}$$

$$\frac{1752.43 \times 100}{2 \times 27} = 3245 \text{ cub. yards} = \text{approx. vol.}$$

$$\text{Corr.} = -\frac{100 \times 18}{12 \times 6 \times 27} \times 94.67 = -91 \text{ cub. yds.}$$

$$3245 - 91 = 3154 \text{ cub. yds.} = \text{exact volume.}$$

The above demonstration of the correction to be applied to the approximate volume, found by averaging end areas, is introduced mainly to give an idea of the amount of that correction. Absolutely level sections are practically unknown, and the error involved in assuming any given sections as truly level will ordinarily be greater than the computed correction. If greater accuracy is required, more points should be obtained in the cross-sectioning, which will generally show that the sections are not truly level.

77. Equivalent sections. When sections are very irregular the following method may be used, especially if great accuracy is not required. The sections are plotted to scale and then a uniform slope line is obtained by stretching a thread so that the undulations are averaged and an *equivalent section* is obtained. The *center depth* (d) and the *slope angle* (a) of this line *can* be obtained from the drawing, but it is more convenient to measure the distances (x_l and x_r) from the center. The area may then be obtained independent of the center depth as follows: Let s = the slope ratio of the side slopes = $\cot \beta = \frac{b}{2a}$. (See Fig. 50.)

Then the

$$\text{Area} = \frac{1}{2}\left(\frac{x_l + x_r}{s}\right)(x_l + x_r) - \frac{x_r}{s}\frac{x_r}{2} - \frac{x_l}{s}\frac{x_l}{2} - \frac{ab}{2}$$

$$= \frac{x_l x_r}{s} - \frac{ab}{2} \qquad (54)$$

Digitized by Google

The true volume, according to the prismoidal formula, of a length of the road measured in this way will be

$$\frac{l}{6}\left[\frac{x_l' x_r'}{s}-\frac{ab}{2}+4\left(\frac{x_l'+x_l''}{2}\,\frac{x_r'+x_r''}{2}\,\frac{1}{s}-\frac{ab}{2}\right)+\frac{x_l'' x_r''}{s}-\frac{ab}{2}\right].$$

If computed by averaging end areas, the approximate volume will be

$$\frac{l}{2}\left[\frac{x_l' x_r'}{s}-\frac{ab}{2}+\frac{x_l'' x_r''}{s}-\frac{ab}{2}\right].$$

Subtracting this result from the true volume, we obtain as the correction

$$\text{Correction}=\frac{l}{6s}(x_l''-x_l')(x_r'-x_r''). \quad . \quad . \quad (55)$$

This shows that if the side distances to either the right or left are equal at adjacent stations the correction is *zero*, and also that if the difference is small the correction is also small and very probably within the limit of accuracy obtainable by that method of cross-sectioning. In fact, as has already been shown in the latter part of § 75, it will usually be a useless

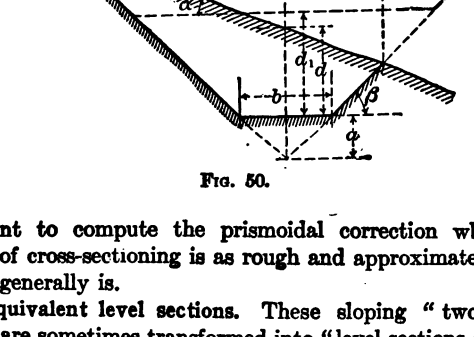


FIG. 50.

refinement to compute the prismoidal correction when the method of cross-sectioning is as rough and approximate as this method generally is.

78. **Equivalent level sections.** These sloping "two-level" sections are sometimes transformed into "level sections of equal

Digitized by Google

area," and the volume computed by the method of level sections (§ 75). But the true volume of a prismoid with sloping ends does not agree with that of a prismoid with equivalent bases and level ends except under special conditions, and when this method is used a correction must be applied if accuracy is desired, although, as intimated before, the assumption that the sections have uniform slopes will frequently introduce greater inaccuracies than that of this method of computation. The following demonstration is therefore given to show the scope and limitations of the errors involved in this much used method.

In Fig. 50, let d_1 be the center height which gives an equivalent level section. The area will equal $(a+d_1)^2 s - \frac{ab}{2}$, which must equal the area given in § 77, $\frac{x_l x_r}{s} - \frac{ab}{2}$. $s = \frac{b}{2a}$.

$$\therefore\ (a+d_1)^2 s = \frac{x_l x_r}{s},$$

$$\text{or} \qquad a+d_1 = \frac{\sqrt{x_l x_r}}{s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

To obtain d_1 directly from notes, given in terms of d and α, we may substitute the values of x_l and x_r given in § 74, which gives

$$a+d_1 = (a+d)\frac{\tan\beta}{\sqrt{\tan^2\beta - \tan^2\alpha}} = \frac{a+d}{\sqrt{1-s^2\tan^2\alpha}}. \quad . \quad (57)$$

The *true* volume of the equivalent section may be represented by

$$\frac{ls}{6}\left[(a+d_1')^2 + 4\left(\frac{a+d_1'}{2} + \frac{a+d_1''}{2}\right)^2 + (a+d_1'')^2\right].$$

From this there should be subtracted the volume of the "grade prism" under the roadbed to obtain the volume of the cut that would be actually excavated, but in the following comparison, as well as in other similar comparisons elsewhere made, the volume of the grade prism invariably cancels out, and so for the sake of simplicity it will be disregarded. This expression for volume may be transposed to

$$\frac{ls}{6}\left[\frac{x_l' x_r'}{s^2} + 4\left(\frac{\sqrt{x_l' x_r'}}{2s} + \frac{\sqrt{x_l'' x_r''}}{2s}\right)^2 + \frac{x_l'' x_r''}{s^2}\right].$$

Digitized by Google

The true volume of the prismoid with sloping ends is (see § 77)

$$\frac{l}{6}\left[\frac{x_l'x_r'}{s}+4\left(\left(\frac{x_l'+x_l''}{2}\right)\left(\frac{x_r'+x_r''}{2}\right)\frac{1}{s}\right)+\frac{x_l''x_r''}{s}\right].$$

The difference of the two volumes

$$=\frac{l}{6s}(x_l'x_r'+x_l''x_r'+x_l'x_r''+x_l''x_r''-x_l'x_r'-2\sqrt{x_l'x_r'x_l''x_r''}-x_l''x_r'')$$

$$=\frac{l}{6s}(\sqrt{x_l'x_r''}-\sqrt{x_l''x_r'})^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

This shows that "equivalent level sections" do *not* in general give the true volume, there being an exception when $x_l'x_r''=x_l''x_r'$. This condition is fulfilled when the slope is uniform, i.e., when $a'=a''$. When this is nearly so the error is evidently not large. On the other hand, if the slopes are inclined in opposite directions the error may be very considerable, particularly if the angles of slope are also large.

79. Three-level sections. The next method of cross-sectioning in the order of complexity, and therefore in the order of

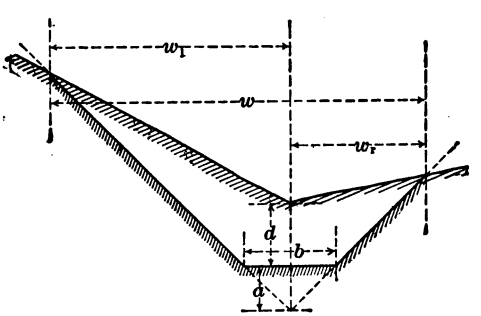


FIG. 51.

accuracy, is the method of three-level sections. The area of the section is $\frac{1}{2}(a+d)(w_r+w_l)-\frac{ab}{2}$, which may be written

Digitized by Google

$\frac{1}{2}(a+d)w-\frac{ab}{2}$, in which $w=w_r+w_l$ If the volume is computed by averaging end areas, it will equal

$$\frac{l}{4}[(a+d')w'-ab+(a+d'')w''-ab]. \quad . \quad . \quad . \quad (59)$$

If we divide by 27 to reduce to cubic yards, we have, when $l=100$,

Vol $_{(,\ldots,)}=\frac{25}{2}(a+d')w'-\frac{25}{4}ab+\frac{25}{2}(a+d'')w''-\frac{25}{4}ab$

For the next section

Vol $_{(,,\ldots,,)}=\frac{25}{2}(a+d'')w''-\frac{25}{4}ab+\frac{25}{2}(a+d''')w'''-\frac{25}{4}ab$.

For a partial station length compute as usual and multiply result by $\frac{\text{length in feet}}{100}$ The prismoidal correction may be obtained by applying Eq 46 to each side in turn For the left side we have

$$\frac{l}{12}[(a+d')-(a+d'')](w_l''-w_l'), \text{ which equals}$$

$$\frac{l}{12}(d'-d'')(w_l''-w_l').$$

For the right side we have, similarly,

$$\frac{l}{12}(d'-d'')(w_r''-w_r').$$

The total correction therefore equals

$$\frac{l}{12}(d'-d'')[(w_l''+w_r'')-(w_l'+w_r')]$$

$$=\frac{l}{12}(d'-d'')(w''-w').$$

Reduced to cubic yards, and with $l=100$,

$$\text{Pris. Corr.}=\frac{25}{81}(d'-d'')(w''-w'). \quad . \quad . \quad . \quad (60)$$

When this result is compared with that given in Eq. 55 there is an apparent inconsistency. If two-level ground is considered as but a special case of three-level ground, it would seem as if

Digitized by Google

the same laws should apply. If, in Eq. 55, $x_r' = x_r''$, and x_l'' is different from x_l', the equation reduces to zero; but in this case d' would also be different from d''; and since $x_l' + x_r'$ would $= w'$, and $x_l'' + x_r'' = w''$ in Eq. 60, $w'' - w'$ would not equal zero and the correction would be some finite quantity and not zero. The explanation lies in the difference in the form and volume of the prismoids, according to the method of the formation of the warped surfaces. If the surface is supposed to be generated by the locus of a line moving parallel to the ends as plane directors and along two *straight* lines lying *in* the side slopes, then $x_l^{\text{mid.}}$ will equal $\frac{1}{2}(x_l' + x_l'')$, and $x_r^{\text{mid.}}$ will equal $\frac{1}{2}(x_r' + x_r'')$, but the profile of the center line will *not* be straight and $d^{\text{mid.}}$ will *not* equal $\frac{1}{2}(d' + d'')$. On the other hand, if the surfaces be generated by *two* lines moving parallel to the ends as plane directors and along a *straight* center line and straight side lines lying in the slopes, a warped surface will be generated each side of the center line, which will have uniform slopes on each side of the center at the two ends and *nowhere else*. This shows that when the upper surface of earthwork is warped (as it generally is), two-level ground should not be considered as a special case of three-level ground. This discussion, however, is only valuable to explain an apparent inconsistency and error. The method of two-level ground should only be used when such refinements as are here discussed are of no importance as affecting the accuracy.

An example is given on the opposite page to illustrate the method of three-level sections.

In the first column of yards

$$210 = \tfrac{2}{3}\tfrac{5}{7}(a+d)w = \tfrac{2}{3}\tfrac{5}{7} \times 7.3 \times 31.1;$$
507, 734, etc., are found similarly;
$$595 = 210 - 61 + 507 - 61;$$
$$448 = \tfrac{4}{1}\tfrac{0}{0}\tfrac{0}{0}(507 - 61 + 734 - 61);$$
$$602 = \tfrac{6}{1}\tfrac{0}{0}\tfrac{0}{0}(734 - 61 + 392 - 61);$$
$$449 = 392 - 61 + 179 - 61.$$

For the prismoidal correction,

$$-20 = \tfrac{2}{3}\tfrac{5}{7}(d' - d'')(w'' - w') = \tfrac{2}{3}\tfrac{5}{7}(2.6 - 8.1)(42.8 - 31.1)$$
$$= \tfrac{2}{3}\tfrac{5}{7}(-5.5)(+11.7).$$

For the next line, $-3 = \tfrac{4}{1}\tfrac{0}{0}\tfrac{0}{0}[\tfrac{2}{3}\tfrac{5}{7}(-2.6)(+8.7)]$, and similarly for the rest. The "F" in the columns of center heights, as well

Digitized by Google

Station.	Center.	Left.	Right.	$a+d$	w	Yards.		$d'-d''$	$w''-w'$	Pris. Corr.	$x_l \sim x_r$	$\frac{V(x_l \sim x_r)}{3R}$	Curv. Corr.*
17	$2.6F$	$\frac{10.6F}{22.9}$	$\frac{0.8F}{8.2}$	7.3	31.1	210					14.7	+1	
18	$8.1F$	$\frac{15.8F}{30.7}$	$\frac{3.4F}{12.1}$	12.8	42.8	507	595	−5.5	+11.7	−20	18.6	+3	+4
+ 40	$10.7F$	$\frac{20.2F}{37.3}$	$\frac{4.8F}{14.2}$	15.4	51.5	734	448	−2.6	+ 8.7	− 3	23.1	+6	+4
19	$6.4F$	$\frac{14.0F}{28.0}$	$\frac{2.1F}{10.1}$	11.1	38.1	392	602	+4.3	−13.4	−11	17.9	+2	+5
20	$3.7F$	$\frac{5.8F}{15.7}$	$\frac{0.2F}{7.3}$	8.4	23.0	179	449	+2.7	−15.1	−13	8.4	+1	+3
										−47			+16

Roadbed, 14′ wide in fill.
Slope 1½ to 1.

Approx. Vol. = 2094
Pris. corr. = 47
True Vol. = 2047 (disregarding curv. corr.)*

$a = \frac{b}{2s} = \frac{14}{3} = 4.7$;

$\frac{25}{27}ab = 61.$

*For the derivation of the curvation correction, see § 93.

Digitized by Google

as in the columns of "right" and "left," are inserted to indicate *fill* for all those points. Cut would be indicated by "*C*."

80. Computation of products. The quantities $\frac{25}{27}(a+d)w$ and $\frac{25}{27}ab$ represent in each case the product of two variable terms and a constant. These products are sometimes obtained from tables which are calculated for all ordinary ranges of the variable terms as arguments. A similar table computed for $\frac{25}{81}(d'-d'')(w''-w')$ will assist similarly in computing the prismoidal correction. Prof. Charles L. Crandall, of Cornell University, is believed to be the first to prepare such a set of tables, which were first published in 1886 in "Tables for the Computation of Railway and Other Earthwork." Another easy method of obtaining these products is by the use of a slide-rule. A slide-rule has been designed by the author to accompany this volume.* It is designed particularly for this special work, although it may be utilized for many other purposes for which slide-rules are valuable. To illustrate its use, suppose $(a+d)=28.2$, and $w=62.4$; then

$$\frac{25}{27}(a+d)w=\frac{28.2\times 62.4}{1.08}.$$

Set 108 (which, being a constant of frequent use, is specially marked) on the sliding scale (B) opposite 282 on the other scale (A), and then opposite 624 on scale B will be found 1629 on scale A, the 162 being read directly and the 9 read by estimation Although strict rules may be followed for pointing off the final result, it only requires a very simple mental calculation to know that the result must be 1629 rather than 162.9 or 16290. For products less than 1000 cubic yards the result may be read directly from the scale; for products between 1000 and 5000 the result may be read directly to the nearest 10

* The first edition of this book was octavo, and a pasteboard slide-rule, especially marked, accompanied each volume. Cutting down the size of the pages to "pocket size" prevents the incorporation of the rule with the present edition. Any slide-rule with a logarithmic unit 22½ inches long will do equally well provided that the 108 mark is specially distinguished for ready use in computing the volume and that the 324 mark is similarly distinguished for use in computing the prismoidal correction.

Digitized by Google

yards, and the tenths of a division estimated. Between 5000 and 10000 yards the result may be read directly to the nearest 20 yards, and the fraction estimated; but prisms of such volume will never be found as simple triangular prisms—at least, an assumption that any mass of ground was as regular as this would probably involve more error than would occur from faulty estimation of fractional parts. Facilities for reading as high as 10000 cubic yards would not have been put on the scale except for the necessity of finding such products as $\frac{2}{3}\frac{5}{7}(9.1\times9.5)$, for example. This product would be read off from the same part of the rule as $\frac{2}{3}\frac{5}{7}(91\times95)$. In the first case the product (80.0) could be read directly to the nearest .2 of a cubic yard, which is unnecessarily accurate. In the other case, the product (8004) could only be obtained by estimating $\frac{4}{2\,0}$ of a division.

The computation for the prismoidal correction may be made similarly except that the divisor is 3.24 instead of 1.08. For example, $\frac{2}{8}\frac{5}{1}(5.5\times11.7)=\frac{5.5\times11.7}{3.24}$. Set the 324 on scale *B* (also specially marked like 108) opposite 55 on scale *A*, and proceed as before.

81. Five-level sections. Sometimes the elevations over each edge of the roadbed are observed when cross-sectioning. These are distinctively termed "five-level sections." If the center, the slope-stakes, and *one* intermediate point on each side (*not* necessarily over the edge of the roadbed) are observed, it is termed an "irregular section." The field-work of cross-sectioning five-level sections is no less than for irregular sections with one intermediate point; the computations, although capable of peculiar treatment on account of the location of the intermediate point, are no easier, and in some respects more laborious; the cross-sections obtained will not in general represent the actual cross-sections as truly as when there is perfect freedom in locating the intermediate point; as it is generally inadvisable or unnecessary to employ five-level sections throughout the length of a road, the change from one method to another adds a possible element of inaccuracy and loses the advantage of uniformity of method, particularly in the notes and *form* of computations. On these accounts the method will not be further developed, except to note that this case, as well as any other, may be solved by dividing the whole prismoid into triangular prismoids, computing the volume by averaging end areas, and computing

Digitized by Google

the prismoidal correction by adding the computed corrections for each elementary triangular prismoid.

82. Irregular sections. In cross-sectioning irregular sections, the distance from the center and the elevation above "grade" of every "break" in the cross-section must be observed. The area of the irregular section may be obtained by computing the area of the trapezoids (*five*, in Fig. 52 and subtracting the two external triangles. For Fig. 52 the area would be

$$\frac{h_l+k_l}{2}(x_l-y_l)+\frac{k_l+d}{2}y_l+\frac{d+j_r}{2}z_r+\frac{j_r+k_r}{2}(y_r-z_r)$$
$$+\frac{k_r+h_r}{2}(x_r-y_r)-\frac{h_l}{2}\left(x_l-\frac{b}{2}\right)-\frac{h_r}{2}\left(x_r-\frac{b}{2}\right).$$

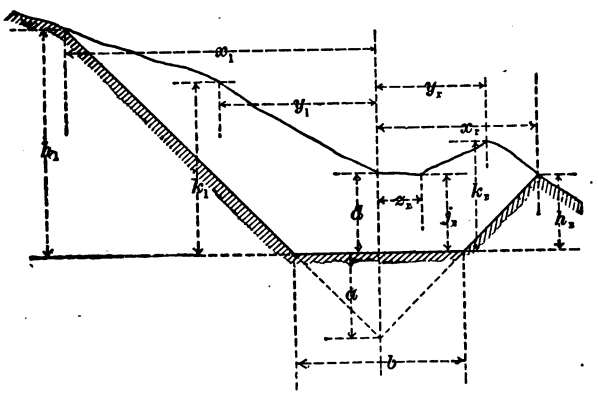


FIG. 52.

Expanding this and collecting terms, of which many will cancel, we obtain

$$\text{AREA}=\frac{1}{2}\Big[x_lk_l+y_l(d-h_l)+x_rk_r+y_r(j_r-h_r)$$
$$+z_r(d-k_r)+\frac{b}{2}(h_l+h_r)\Big]. \quad . \quad . \quad . \quad (61)$$

An examination of this formula will show a perfect regularity in its formation which will enable one to write out a similar formula for any section, no matter how irregular or how

Digitized by Google

many points there are, without any of the preliminary work. The formula may be expressed in words as follows:

Area equals one-half the sum of products obtained as follows:

the distance to each slope-stake times the height above grade of the point next inside the slope-stake;

the distance to each intermediate point in turn times the height of the point just inside minus the height of the point just outside;

finally, one-half the width of the roadbed times the sum of the slope-stake heights.

If one of the sides is perfectly regular from center to slope-stake, it is easy to show that the rule holds literally good. The "point next inside the slope-stake" in this case is the center; the intermediate terms for that side vanish. The *last term* must always be used. The rule holds good for three-level sections, in which case there are three terms, which may be reduced to two. Since these two terms are both variable quantities for each cross-section, the special method, given in § 78, in which one term $\left(\frac{ab}{2}\right)$ is a constant for all sections, is preferable. In the general method, each intermediate "break" adds another term.

83. Volume of an irregular prismoid. This is obtained by computing first the approximate volume by "averaging end areas" or by multiplying the length by the half sum of the end areas, as computed from Eq. (61). In other words, the Approx. volume $= \frac{100}{27} \times \frac{1}{2}$ (area′ + area″). But since each area equals *one-half* the sum of products of width times height (see Eq. (61)) we may say that

$$\text{Approx. volume} = \frac{25}{27} (\text{summation of } \textit{width} \text{ times } \textit{height}) \quad . \quad (62)$$

the terms of width times height being like those found within the bracket of Eq. (61).

As before, for partial station lengths, multiply the result by (length in feet ÷ 100). There will be no constant subtractive term, $\frac{25}{27} ab$, as in § 79.

The correction to this approximate volume is found by considering that *for the purpose of this correction only* the end sections are considered as "three-level" sections and the correction is computed by applying Eq. (60).

Digitized by Google

84. Numerical example; irregular sections; volume with approximate prismoidal correction. Assume the earthwork notes as given below, where the roadbed is 18 feet wide in cut and the slope is 1½ to 1. Note that the stations read UP the page and that when the surveyor is looking ahead along the line the several combinations of *heights* and *distances out* have approximately the same relative position on the note book as they have on the ground. For example, beginning at the bottom line (Sta. 16) the combination $\frac{8.9c}{22.4}$ means that the extreme left-hand point of that section (the "slope stake") is 22.4 feet horizontally from the center and that it is 8.9 feet above the required roadbed. The cut (*c*) *would be* 8.9 feet to reach the roadbed, but of course the actual cutting is zero at the slope stake. The next point is 12.0 feet horizontally from the center and 7.6 feet above the roadbed. The cut at the center is 6.8 feet. The combinations of dimensions on the right-hand side are to be interpreted similarly.

Sta.	Center { cut or fill.	Left.			Right.	
19	0.6*c*	$\frac{3.6c}{14.4}$			$\frac{0.1c}{4.2}$	$\frac{0.4c}{9.6}$
18	2.3*c*	$\frac{4.2c}{15.3}$	$\frac{6.8c}{8.4}$	$\frac{3.2c}{5.2}$		$\frac{1.2c}{10.8}$
17	7.6*c*	$\frac{8.2c}{21.3}$	$\frac{10.2c}{17.4}$	$\frac{8.0c}{6.1}$		$\frac{4.2c}{15.3}$
+42	10.2*c*	$\frac{12.2c}{27.3}$		$\frac{12\ 6c}{8.2}$	$\frac{6.2c}{7.5}$	$\frac{8.4c}{21.6}$
16	6.8*c*	$\frac{8.9c}{22.4}$		$\frac{7.6c}{12.0}$	$\frac{3.2c}{4.1}$	$\frac{2.6c}{12.9}$

The numerical computation is greatly facilitated by a systematic form as given below. For Sta. 16, the first term is "the distance to the left slope stake" (22.4) times "the height above grade of the point next inside" (the height being 7.6), and we place this pair of figures in the columns of "width" and "height." The "distance to the point next inside" is 12.0 and the "height of the point just inside (6.8) minus the

Digitized by Google

height of the point just outside" (8.9) equals (−2.1) and these are the next pair of widths and heights. Taking $\frac{25}{27}$ of the product of each pair of numbers we have the numbers in the first column of "yards." The sum of all these numbers in the first and second groups multiplied by $\frac{42}{100}$ (that section being only 42 feet long) equals 378 cubic yards, the volume by averaging end areas. The determination of center heights and total widths and the application of Eq. (60), to obtain the approximate prismoidal correction, is self-evident.

VOLUME OF IRREGULAR PRISMOID, WITH APPROXIMATE PRISMOIDAL CORRECTION.

Sta.	W'th	H'ght	Yards.		Cen. Height.	Total width	$d'-d''$	$w''-w'$	Approx. pris.corr.
	22.4	7.6	158		+6.8	35.3			
	12.0	−2.1	−23						
16	12.9	3.2	40						
	4.1	4.2	16						
	9.0	11.5	96						
	27.3	12.6	319		+10.2	48.9	−3.4	+13.6	−14
	8.2	−2.0	−15						
+42	21.6	6.2	124						
	7.5	1.8	13						
	9.0	20.6	172	378					(−6)
	21.3	10.2	201		+ 7.6	36.6	+2.6	−12.3	−10
	17.4	−0.2	− 3						
17	6.1	−2.6	−14						
	15.3	7.6	107						
	9.0	12.4	103	584					(−6)
	15.3	6.8	95		+ 2.3	26.1	+5.3	−10.5	−17
	8.4	−1.0	− 7						
18	5.2	−4.5	−22						
	10.8	2.3	23						
	9.0	5.4	45	528					(−17)
	14.4	0.6	8		+ 0.6	24.0	+1.7	−2.1	−1
	9.6	0.1	1						
19	4.2	0.2	1						
	9.0	4.0	33	177					(−1)
									−30

Approx. volume = 1667
Approx. pris. corr. = −30

Corrected volume = 1637 cubic yards

85. Magnitude of the probable error of this method. In previous editions of this work, methods were given for computing the mathematically exact volume of a prismoid whose ends coincide with the "irregular sections" as measured, and

Digitized by Google

whose upper surfaces are assumed to coincide with the actual surface of the ground. As in the previous methods, the "approximate volume" is computed by averaging end areas and then a correction is applied. If the end sections have the same number of intermediate points on each side, and if it can be assumed that the corresponding lines in each section are connected by plane or warped surfaces, which coincide with the surface of the ground, then the mathematically exact or "true" correction may be obtained by dividing the volume into elementary triangular prismoids, finding the correction for each and adding the results. Although such a method appears very complicated, it is readily possible to develop a law by means of which the true prismoidal correction may be written out (similarly to writing out the formula for the area, Eq. (61)) without any preliminary calculation. Such a law has a mathematical fascination, but it should be remembered that when the ground surface is so broken up that the cross-sections are "irregular" it is in general correspondingly rough and irregular between the cross-sections, especially when those sections are 100 feet apart. It is also true that the cross-sections do *not* usually have the same number of intermediate points on corresponding sides of the center. In such a case, unless the actual form of the ground between the cross-sections is observed and measured, the exact method cannot be used. An extra point in one cross-section implies an extra ridge (or hollow) which "runs out" or disappears by the time the adjoining section is reached. Theoretically a cross-section should be taken at the point where such a ridge or hollow runs out. In general this point will not be at an even 100-foot station. The attempt to compute the exact prismoidal correction usually gives merely a false appearance of extreme accuracy to the work which is not justified by the results. It should not be forgotten that it is readily possible to spend an amount of time on the surveying and computing which is worth more than the few cubic yards of earth which represents the additional accuracy of the more precise method. The accuracy of the office computation should be kept proportionate to the accuracy of the cross-sectioning in the field. The discussion of the magnitude of the prismoidal correction in §§ 72–79 shows that it is small except when the two ends of the prismoid are very dissimilar. The dissimiliarity between the two ends of a prismoid would be substantially the

Digitized by Google

same whether the ends were actually "irregular" or had "three-level" sections, which for each end had the same slope stakes and center heights as the irregular sections. Experience proves that the approximate prismoidal correction, computed by considering the ground as three-level, is so nearly equal to the true prismoidal correction that the difference is perhaps no greater than the *probable* difference between the true volume of earth and the volume of the geometrical prismoid which is assumed to represent that volume. The experienced surveyor will take his cross-sections at such places and so close together that the warped surfaces joining the sections will lie very nearly in the surface or at least will so average the errors that they will substantially neutralize each other.

86. Numerical illustration of the accuracy of the approximate rule. The "true" prismoidal correction for the numerical case given in § 84 was computed by the method outlined above, and on the basis of certain figures as to the vanishing of the ridges and valleys found in one section and not found in the adjacent sections. The various quantities for the volumes between the cross-sections have been tabulated as shown.

	1	2	3	4	5	6	7
Sections.	Approx. vol. by averaging end areas.	True prismoidal correction.	True volume.	Approx. pris. corr. on basis of three-level ground.	Error: Col. 4 − col. 2.	Approx. vol. computed from center and side heights *only*.	Error: Col. 6 − col. 3.
16.....16+42	378	− 5	373	− 6	−1	396	−23
16+42..17	584	− 3	581	− 6	−3	577	+ 4
17.....18	528	−16	512	−17	−1	463	+49
18.....19	177	− 3	174	− 1	+2	147	+27
	1667	−27	1640	−30	−3	1583	+57

There has also been shown in the last two columns the error involved if the "intermediate points" had been ignored in the cross-sectioning. From the tabular form we may learn that

1. The *differences* between the "true" and approximate corrections is so small that it is *probably* swallowed up by errors resulting from inaccurate cross-sectioning.

2. The error which would have been involved in ignoring the intermediate points is so very large in comparison with

Digitized by Google

the other corresponding errors that (although it proves nothing absolutely definite, being an individual case) the *probabilities* of the relative error from these sources are clearly indicated.

87. Cross-sectioning irregular sections. The slope stake should preferably be determined first, and then the "breaks" between the slope stake and the center. When, as is usual, the ground is not even between the cross-section just taken and the section at the next 100-foot station, a point should be selected for a cross-section such that the lines to the previous section should coincide with the actual surface of the ground as closely as the accuracy of the work demands. § 94 gives a numerical illustration of the magnitude of some of these errors. Although it is possible for a skillful surveyor to so choose his cross-sections in rough and irregular ground that the positive and negative errors will nearly balance, it requires exceptional skill. Frequently the work may be simplified by computing separately the volume of a mound or pit, the existence of which has been ignored in the regular cross-sectioning.

88. Side-hill work. When the natural slope cuts the roadbed there is a necessity for both cut and fill at the same cross-section.

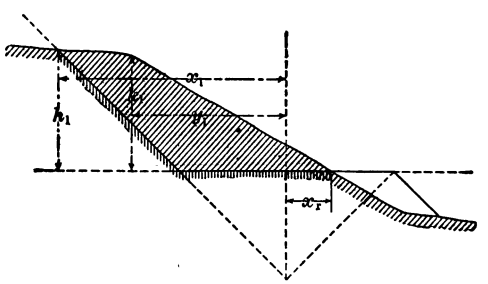


FIG. 53.

When this occurs the cross-sections of both cut and fill are often so nearly triangular that they may be considered as such without great error, and the volumes may be computed separately as triangular prismoids without adopting the more elaborate form of computation so necessary for complicated irregular sections. When the ground is too irregular for this the best plan is to

Digitized by Google

follow the uniform system. In computing the cut, as in Fig. 53, the left side would be as usual; there would be a small center cut and an ordinate of zero at a short distance to the right of the center. Then, *ignoring the fill*, and applying Eq. 61 strictly, we have two terms for the left side, one for the right, and the term involving $\frac{1}{2}b$, which will be $\frac{1}{2}bh_l$ in this case, since $h_r=0$, and the equation becomes

$$\text{Area}=\tfrac{1}{2}[x_lk_l+y_l(d-h_l)+x_rd+\tfrac{1}{2}bh_l].$$

The area for fill may also be computed by a strict application of Eq. 61, but for Fig. 54 all distances for the left side are zero and the elevation for the first point out is zero. d also must be

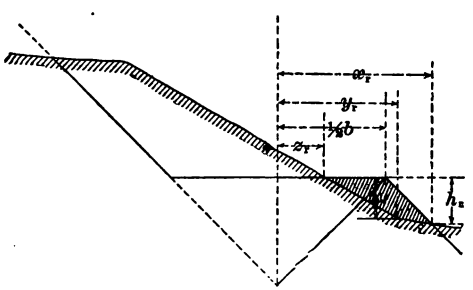


FIG. 54.

considered as zero. Following the rule, § 81, literally, the equation becomes

$$\text{Area}_{(\text{Fill})}=\tfrac{1}{2}[x_rk_r+y_r(o-h_r)+z_r(o-k_r)+\tfrac{1}{2}b(o+h_r)],$$

which reduces to

$$\tfrac{1}{2}[x_rk_r-y_rh_r-z_rk_r+\tfrac{1}{2}bh_r].$$

(*Note* that x_r, h_r, etc., have different significations and values in this and in the preceding paragraphs.) The "terminal pyramids" illustrated in Fig. 40 are instances of side-hill work for very short distances. Since side-hill work always implies *both* cut and fill at the same cross-section, whenever either the cut or fill disappears and the earthwork becomes wholly cut or wholly fill, that point marks the end of the "side-hill work." and a cross-section should be taken at this point.

Digitized by Google

89. Borrow-pits. The cross-sections of borrow-pits will vary not only on account of the undulations of the surface of the

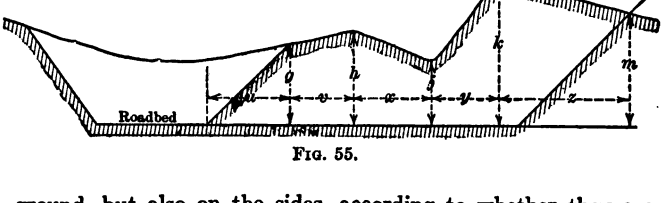


FIG. 55.

ground, but also on the sides, according to whether they are made by widening a convenient cut (as illustrated in Fig. 55) or simply by digging a pit. The sides should always be properly sloped and the cutting made cleanly, so as to avoid unsightly roughness. If the slope ratio on the right-hand side (Fig. 55) is s, the area of the triangle is $\frac{1}{2}sm^2$. The area of the section is $\frac{1}{2}[ug+(g+h)v+(h+j)x+(j+k)y+(k+m)z-sm^2]$. If all the horizontal measurements were referred to one side as an origin, a formula similar to Eq. 61 could readily be developed, but little or no advantage would be gained on account of any simplicity of computation. Since the *exact* volume of the earth borrowed is frequently necessary, the prismoidal correction should be computed; and since such a section as Fig. 55 does not even approximate to a three-level section, the method suggested in § 83 cannot be employed. It will then be necessary to employ the more exact method of dividing the volume into triangular prismoids and taking the summation of their corrections, found according to the general method of § 72.

90. Correction for curvature. The volume of a solid, generated by revolving a plane area about an axis lying in the plane but outside of the area, equals the product of the given area times the length of the path of the center of gravity of the area. If the centers of gravity of all cross-sections lie in the center of the road, where the length of the road is measured, there is absolutely no necessary correction for curvature. If all the cross-sections in any given length were exactly the same and therefore had the same eccentricity, the correction for curvature would be very readily computed according to the above principle. But when both the areas and the eccentricities vary from point to point, as is generally the case, a theoretically exact

Digitized by Google

solution is quite complex, both in its derivation and application. Suppose, for simplicity, a curved section of the road, of uniform cross-sections and with the center of gravity of every cross-section at the same distance e from the center line of the road. The length of the path of the center of gravity will be to the length of the center line as $R \pm e : R$. Therefore we have *True vol.*: *nominal vol.* $:: R \pm e : R$. $\therefore$ *True vol.* $= lA\frac{R \pm e}{R}$ for a volume of uniform area and eccentricity. For any other area and eccentricity we have, similarly, *True vol.'* $= lA'\frac{R \pm e'}{R}$. This shows that the effect of curvature is the same as increasing (or diminishing) the area by a quantity depending on the area and eccentricity, the increased (or diminished) area being found by multiplying the actual area by the ratio $\frac{R \pm e}{R}$. This being independent of the value of l, it is true for infinitesimal lengths. If the eccentricity is assumed to vary uniformly between two sections, the *equivalent area* of a cross-section located midway between the two end cross-sections would be $A_m\frac{\left(R \pm \frac{e' + e''}{2}\right)}{R}$. Therefore the volume of a solid which, when straight, would be $\frac{l}{6}(A' + 4A_m + A'')$, would then become

$$\textit{True vol.} = \frac{l}{6R}\left[A'(R \pm e') + 4\,A_m\left(R \pm \frac{e' + e''}{2}\right) + A''(R \pm e'')\right].$$

Subtracting the nominal volume (the true volume when the prismoid is straight), the

$$\textit{Correction} = \pm\frac{l}{6R}\left[(A' + 2A_m)e' + (2A_m + A'')e''\right]. \quad . \quad (63)$$

Another demonstration of the same result is given by Prof. C. L. Crandall in his "Tables for the Computation of Railway and other Earthwork," in which is obtained by calculus methods the summation of elementary volumes having variable areas with variable eccentricities. The exact application of Eq. 63 requires that A_m be known, which requires laborious computa-

Digitized by Google

tions, but no error worth considering is involved if the equation is written approximately

$$Curv.\ corr. = \frac{l}{2R}(A'e' + A''e''), \quad . \quad . \quad . \quad . \quad (64)$$

which is the equation generally used. The approximation consists in assuming that the difference between A' and A_m equals the difference between A_m and A'' but with opposite sign. The error due to the approximation is always utterly insignificant.

91. Eccentricity of the center of gravity. The determination of the true positions of the centers of gravity of a long series of irregular cross-sections would be a very laborious operation, but fortunately it is generally sufficiently accurate to consider the cross-sections as three-level ground, or, for side-hill work, to

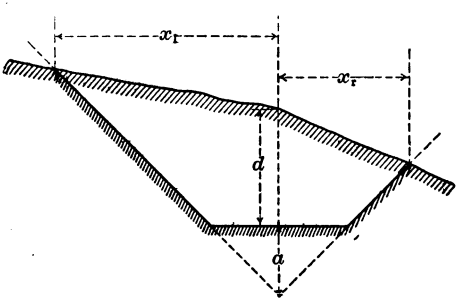


FIG. 56.

be triangular, *for the purpose of this correction.* The eccentricity of the cross-section of Fig. 56 (including the grade triangle) may be written

$$e = \frac{\frac{(a+d)x_l}{2}\frac{x_l}{3} - \frac{(a+d)x_r}{2}\frac{x_r}{3}}{\frac{(a+d)x_l}{2} + \frac{(a+d)x_r}{2}} = \frac{1}{3}\frac{x_l^2 - x_r^2}{x_l + x_r} = \frac{1}{3}(x_l - x_r). \quad . \quad (65)$$

The side toward x_l being considered positive in the above demonstration, if $x_r > x_l$, e would be negative, i.e., the center of gravity would be on the right side. Therefore, for three-level

Digitized by Google

ground, the correction for curvature (see Eq. 64) may be written

$$Correction = \frac{l}{6R}[A'(x_l' - x_r') + A''(x_l'' - x_r'')].$$

Since the approximate volume of the prismoid is

$$\frac{l}{2}(A + A') = \frac{l}{2}A' + \frac{l}{2}A'' = V' + V'',$$

in which V' and V'' represent the number of cubic yards corresponding to the area at each station, we may write

$$Corr.\ in\ cub.\ yds. = \frac{1}{3R}[V'(x_l' - x_r') + V''(x_l'' - x_r'')]. \quad . \quad (66)$$

It should be noted that the value of e, derived in Eq. 65, is the eccentricity of the whole area including the triangle under the roadbed. The eccentricity of the true area is greater than this and equals

$$e \times \frac{\text{true area} + \frac{1}{2}ab}{\text{true area}} = e_1.$$

The required quantity ($A'e'$ of Eq. 64) equals *true area* $\times e_1$ which equals (*true area* $+ \frac{1}{2}ab) \times e$. Since the value of e is very simple, while the value of e_1 would, in general, be a complex quantity, it is easier to use the simple value of Eq. 65 and add $\frac{1}{2}ab$ to the area. Therefore, in the case of three-level ground the subtractive term $\frac{8}{27}ab$ (§ 79) should *not* be subtracted in computing this correction. For irregular ground, when computed by the method given in §§ 82 and 83, which does not involve the grade triangle, a term $\frac{8}{27}ab$ must be *added* at every station when computing the quantities V' and V'' for Eq. 66.

It should be noted that the factor $1 \div 3R$, which is constant for the length of the curve, may be computed with all necessary accuracy and without resorting to tables by remembering that

$$R = \frac{5730}{\text{degree of curve}}.$$

Since it is useless to attempt the computation of railroad earthwork closer than the nearest cubic yard, it will frequently

Digitized by Google

be possible to write out all curvature corrections by a simple mental process upon a mere inspection of the computation sheet. Eq. 66 shows that the correction for each station is of the form $\frac{V(x_l - x_r)}{3R}$. $3R$ is generally a large quantity—for a 6° curve it is 2865. $(x_l - x_r)$ is generally small. It may frequently be seen by inspection that the product $V(x_l - x_r)$ is roughly twice or three times $3R$, or perhaps less than half of $3R$, so that the corrective term for that station may be written 2, 3, or 0 cubic yards, the fraction being disregarded. For much larger absolute amounts the correction must be computed with a correspondingly closer percentage of accuracy.

The algebraic sign of the curvature correction is best determined by noting that the center of gravity of the cross-section is on the right or left side of the center according as x_r is greater or less than x_l, and that the correction is *positive* if the center of gravity is on the *outside* of the curve, and *negative* if on the *inside*.

It is frequently found that x_l is uniformly greater (or uniformly less) than x_r throughout the length of the curve. Then the curvature correction for each station is uniformly positive or negative. But in irregular ground the center of gravity is apt

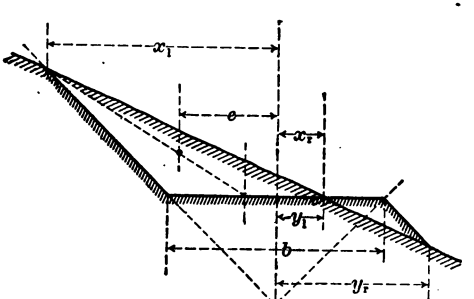


FIG. 57.

to be irregularly on the outside or on the inside of the curve, and the curvature correction will be correspondingly positive or negative. If the curve is to the *right*, the correction will be positive or negative according as $(x_l - x_r)$ is positive or negative; if the curve is to the *left*, the correction will be positive or nega-

Digitized by Google

tive according as $(x_r - x_l)$ is positive or negative. Therefore when computing curves to the *right* use the form $(x_l - x_r)$ in Eqs. 66 and 68; when computing curves to the *left* use the form $(x_r - x_l)$ in these equations; the algebraic sign of the correction will then be strictly in accordance with the results thus obtained.

92. Center of gravity of side-hill sections. In computing the correction for side-hill work the cross-section would be treated as triangular unless the error involved would evidently be too great to be disregarded. The center of gravity of the triangle lies on the line joining the vertex with the middle of the base and at $\frac{1}{3}$ of the length of this line from the base. It is therefore equal to the distance from the center to the foot of this line plus $\frac{1}{3}$ of its horizontal projection. Therefore

$$e = \left[\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right] + \frac{1}{3}\left[x_l - \left(\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right)\right]$$

$$= \frac{b}{4} - \frac{x_r}{2} + \frac{x_l}{3} - \frac{b}{12} + \frac{x_r}{6}$$

$$= \frac{b}{6} + \frac{x_l}{3} - \frac{x_r}{3}$$

$$= \frac{1}{3}\left[\frac{b}{2} + (x_l - x_r)\right] \quad \ldots\ldots\ldots\ldots \quad (67)$$

By the same process as that used in § 91 the correction equation may be written

$$\text{Corr. in cub. yds.} = \frac{1}{3R}\left[V'\left(\frac{b}{2} + (x_l' - x_r')\right) + V''\left(\frac{b}{2} + (x_l'' - x_r'')\right)\right]. \quad (68)$$

It should be noted that since the grade triangle is not used in this computation the volume of the grade prism is *not* involved in computing the quantities V' and V''.

The eccentricities of cross-sections in side-hill work are *never* zero, and are frequently quite large. The total volume is generally quite small. It follows that the correction for curvature is generally a vastly larger proportion of the total volume than in ordinary three-level or irregular sections.

If the triangle is wholly to one side of the center, Eq. 67 can still be used. For example, to compute the eccentricity of the triangle of fill, Fig. 57, denote the two distances to the slope-

Digitized by Google

stakes by y_r and $-y_l$ (note the minus sign). Applying Eq. 67 literally (noting that $\frac{b}{2}$ must here be considered as negative in order to make the notation consistent) we obtain

$$e=\frac{1}{3}\left[-\frac{b}{2}+(-y_l-y_r)\right],$$

which reduces to

$$e=-\frac{1}{3}\left[\frac{b}{2}+y_l+y_r\right]. \quad . \quad . \quad . \quad . \quad . \quad (69)$$

As the algebraic signs tend to create confusion in these formulæ, it is more simple to remember that for a triangle lying on *both* sides of the center e is always numerically equal to $\frac{1}{3}\left[\frac{b}{2}+(x_l\sim x_r)\right]$, and for a triangle entirely on one side, e is numerically equal to $\frac{1}{3}\left[\frac{b}{2}+$the numerical *sum* of the two distances out]. The algebraic sign of e is readily determinable as in § 91.

93. Example of curvature correction. Assume that the fill in § 79 occurred on a 6° curve to the *right*. $\frac{1}{3R}=\frac{1}{2865}$. The quantities 210, 507, etc., represent the quantities V', V'', etc., since they include in each case the 61 cubic yards due to the grade prism. Then

$$\frac{V(x_l\sim x_r)}{3R}=\frac{210(22.9-8.2)}{2865}=\frac{3101.7}{2865}=+1.$$

The sign is plus, since the center of gravity of the cross-section is on the left side of the center and the road curves to the right, thus making the true volume larger. For Sta. 18 the correction, computed similarly, is +3, and the correction for the whole section is 1+3=4. For Sta. 18+40 the correction is computed as 6 yards. Therefore, for the 40 feet, the correction is $\frac{40}{100}(3+6)=3.6$, which is called 4. Computing the others similarly we obtain a total correction of +16 cubic yards.

Digitized by Google

94. Accuracy of earthwork computations. The preceding methods give the *precise volume* (except where approximations are distinctly admitted) of the prismoids which are *supposed* to represent the volume of the earthwork. To appreciate the accuracy necessary in cross-sectioning to obtain a given accuracy in volume, consider that a fifteen-foot length of the cross-section, which is assumed to be straight, really sags 0.1 foot, so that the cross-section is in error by a triangle 15 feet wide and 0.1 foot high. This sag 0.1 foot high would hardly be detected by the eye, but in a length of 100 feet in each direction it would make an error of volume of 1.4 cubic yards in *each* of the two prismoids, assuming that the sections at the other ends were perfect. If the cross-sections at both ends of a prismoid were in error by this same amount, the volume of that prismoid would be in error by 2.8 cubic yards if the errors of area were both plus or both minus. If one were plus and one minus, the errors would neutralize each other, and it is the compensating character of these errors which permits any confidence in the results as obtained by the usual methods of cross-sectioning. It demonstrates the utter futility of attempting any closer accuracy than the nearest cubic yard. It will thus be seen that if an error really exists at *any* cross-section it involves the prismoids on *both* sides of the section, even though all the other cross-sections are perfect. As a further illustration, suppose that cross-sections were taken by the method of slope angle and center depth (§ 74), and that a cross-section, assumed as uniform, sags 0.4 foot in a width of 20 feet. Assume an equal error (of same sign) at the other end of a 100-foot section. The error of volume for that one prismoid is 38 cubic yards.

The computations further assume that the warped surface, passing through the end sections, coincides with the surface of the ground. Suppose that the cross-sectioning had been done with mathematical perfection; and, to assume a simple case, suppose a sag of 0.5 foot between the sections, which causes an error equal to the volume of a pyramid having a base of 20 feet (in each cross-section) times 100 feet (between the cross-sections) and a height of 0.5 foot. The volume of this pyramid is $\frac{1}{3}(20\times 100)\times 0.5 = 333$ cub. ft. $= 12$ cub. yds. And yet this sag or hump of 6 inches would generally be utterly unnoticed, or at least disregarded.

When the ground is very rough and broken it is sometimes

Digitized by Google

practically impossible, even with frequent cross-sections, to locate warped surfaces which will closely coincide with all the sudden irregularities of the ground. In such cases the computations are necessarily more or less approximate and dependence must be placed on the compensating character of the errors.

95. Approximate computations from profiles. When a "paper location" has been laid out on a topographical map having contours, it is possible to compute approximately the amount of earthwork required by some very simple and rapid calculations. A profile may be readily drawn by noting the intersections of the proposed center line with the various contours and plotting the surface line on profile paper. Drawing the grade-line on the profile, the depth of cut or fill may be scaled off at any point. When it is only desired to obtain

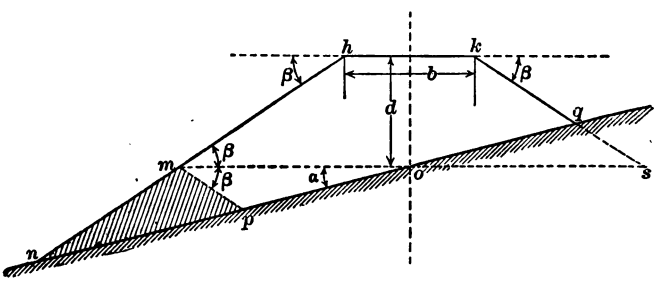


FIG. 58.

very quickly an approximate estimate of the amount of earthwork required on a suggested line, it may be done by the method described in § 75, or by the use of Table XXXIII. But the assumption that the surface of the ground at each cross-section is level invariably has the effect that the estimated volumes are not as large as those actually required. The difference between the "level section" *hkms* and the actual slope section *hknq* equals the difference between the triangles *mon* and *oqs*, and this difference equals the shaded area *mpn*. The excess volume is proportional to the area of the triangle *mpn*. This area may be expressed by the formula,

$$\text{Area } mpn = 2(\tfrac{1}{2}b + d \cot \beta)^2 \frac{\sin^2\alpha \sin\beta \cos\beta}{\cos 2\alpha - \cos 2\beta}.$$

Digitized by Google

The percentage of this excess area to the nominal area *hkms* therefore depends on the dimensions b and d and the angles α and β. A solution of this equation for ninety different combinations of various numerical values for these four variables is included in Table XXXIII for the purpose of making corrections. A study of this correction table points conclusively to the following laws, a thorough understanding of which will enable an engineer to appreciate the degree of accuracy which is attainable by this approximate method:

(a) Increasing the *width* of the roadbed (b), the other three factors remaining constant, *increases* the percentage of error, but the increase is comparatively small.

(b) Increasing the *depth* of cut or fill (d), *decreases* the percentage of error, but the decrease is almost insignificant.

(c) Increasing the angle of the side slopes (β) *decreases* the percentage of error, the decrease being very considerable.

(d) Increasing the angle of the slope of the ground (α), *increases* the percentage of error, the percentage rapidly increasing to infinity as the value of α approaches that of β. This is another method of stating the fact that α must always be less than β and, practically, must be considerably less, so that the slope stake shall be within a reasonable distance from the center.

Since the above value for the corrective area is a function of the angle α, which is usually variable and whose value is frequently known only approximately, it is useless to attempt to apply the correction with great precision, and the following rules will usually be found amply accurate, considering the probable lack of precision in the data used.

1. For embankments or cuts, having a slope of 1.5:1, and with a surface slope of 5° (nearly 9%) the excess of true area over nominal area is *about* 2%. There is only a slight variation from this value for all ordinary depths (d) and widths (b) of roadbed. Therefore the nominal volume would be about 2% too *small*. On the other hand, the effect of the prismoidal correction is such that, even with truly level sections, the nominal volume is too *large*. See §§ 75 and 76. The amount of the prismoidal correction depends on the differences between successive center depths. In the very ordinary numerical case given in § 76, the correction was nearly 3%, which more than neutralizes the error due to surface slope. Therefore in

Digitized by Google

many cases on slightly sloping ground the error due to the surface slope will so nearly neutralize the prismoidal correction that the quantities taken directly from the tables (without correction for either cause) will equal the true volume with as close an approach to accuracy as the precision of the surveying will permit.

2. For a cut with a slope of 1:1, and with a surface slope of 5° the error is about 1%. This will be neutralized by still smaller prismoidal corrections. Therefore, for surface slopes of 5° or less, no allowance should be made for this error unless the prismoidal correction is also considered.

3. When the surface slope is 10° (nearly 18%) the error for a 1.5:1 slope is from 7% to 10% and for a 1:1 slope from 3% to 5%.

4. For a 30° surface slope and 1.5:1 side slopes the excess volume is three or four times the nominal volume. Such a steep surface slope implies the probability of "side-hill work" to which the above corrective rules are not applicable. When the surface slopes are very steep careful work must be done to avoid excessive error. For a 1:1 side slope, the errors are from 50% to 80%.

A still closer approximation, especially for the steeper surface slopes, may be obtained by using, directly or by interpolation, figures from the corrective tabular form which forms part of Table XXXIII. Unless the surface slope angle is known accurately (especially when large) no great accuracy in the final result is possible. Close accuracy would also require the determination of the prismoidal correction. But if such close accuracy is deemed essential, it can be most easily obtained by accurate cross-sectioning at each station and the adoption of other methods of computation—such as are given in §§ 83 and 84.

When the contours have been drawn in for a sufficient distance on either side to include the position of both slope stakes at every station, as will usually be the case, cross-sections may be obtained by drawing lines on the map at each station perpendicular to the center line—see Fig. 4. The intersection of these lines with the contours will furnish the distances for drawing on cross-section paper the transverse profile at each station. Drawing on the same cross-section the lines representing the roadbed and the side slopes, the cross-section of

Digitized by Google

cut (or fill) is complete and its area may be obtained by scaling from the cross-section paper. If the contours have been located on the map with sufficient accuracy, such a method will determine the cross-sectional area very closely. When cross-sections have been taken with a wye- or hand-level, as described in § 12, the cross-sections as plotted will probably be more accurate than when the contours are run in from points determined by the stadia method. In fact this semi-graphical method is frequently used, in place of the purely numerical methods described in previous sections, to make final estimates of the volume of earthwork.

As a numerical example, an assumed location line was laid out on the contours given in Fig. 4. The volume of cut, as determined by Table XXXIII for a roadbed 20 feet wide, with side slopes of 1:1, was 5746 cubic yards. The surface slope varied from 3° to 11°. Computing the corrections by a careful interpolation from the corrective table, the total correction was found to be 128 cubic yards, or an average of a little over 2%. On the other hand the negative prismoidal correction amounts to 72 cubic yards, which leaves a net correction of 56 cubic yards—about 1%. It so happens that in this case a correction for curvature would tend further to wipe out this correction. These figures merely verify numerically the general conclusions stated above, although it should not be forgotten that in individual cases the figures taken from Table XXXIII require ample correction.

The following approximate rule, for which the author is indebted to Mr. W. H. Edinger, is exceedingly useful when it is desired to rapidly determine the approximate volume of earthwork between two points along the road. Its great merit lies in the fact that it only means the memorizing of a comparatively simple rule which will make it possible to make such computations in the field, without the use of tables. The rule is based on the fact that the area of any level section equals $bd + sd^2$; and therefore,

$$\Sigma(\text{vol.}) = (b\,\Sigma d + s\,\Sigma d^2)\frac{L}{27},$$

in which L is usually 100 feet. For strict accuracy this would only be the volume provided the total length was an even number of hundred feet, and the various values of d represented

Digitized by Google

the depths which were uniform for hundred foot sections. It makes no allowance for the comparatively large prismoidal error of the pyramidal and wedge-shaped sections usually found at each end of a cut or fill, but where an approximate estimate is desired, in which this inaccuracy may be neglected, the method is very useful. The method of applying this rule without tables may best be illustrated by a simple numerical example. Assume that the levels on a stretch of fairly level ground, which is about 500 feet long, have been taken, the depths being taken at points 100 feet apart, the first and last points being about 40 or 50 feet from the ends of the cut, or fill. The depths are as given in the first column in the tabular form below; the slope is 1.5:1, and the breadth (b) is 14 feet.

d	d^2
1.6	2.56
2.8	7.84
4.5	20.25
3.1	9.61
0.9	.81
$\Sigma d =$ 12.9	$\Sigma d^2 = 41.07$
• 14	20.53
$b\Sigma d = 180.6$	$s\Sigma d^2 = 61.60$
61.60	
242.2	

$24220 \div 27 = 897$ cubic yards.

The 180.6 is the $b\Sigma d$ and the 61.6 is $s\Sigma d^2$; adding these and moving the decimal point two places to multiply by 100, we only have to divide by 27 to obtain the value in cubic yards. Although the above rule requires more work than the employment of earthwork tables, yet it is a very convenient method of estimating the approximate volume of a short section of carthwork when no tables are at hand.

96. Shrinkage of earthwork. The statistical data indicating the amount of shrinkage is very conflicting, a fact which is probably due to the following causes:

1. The various kinds of earthy material act very differently as respects shrinkage. There is a great lack of uniformity in

Digitized by Google

the *classification of earths* in the tests and experiments which have been made.

2. Very much depends on the *method* of forming an embankment (as will be shown later). Different reports have been based on different methods—often without mention of the method.

3. An embankment requires considerable *time* to shrink to its final volume, and therefore much depends on the time elapsed between construction and the measurement of what is supposed to be the settled volume.

4. A soft subsoil will frequently settle under the weight of a high embankment and apparently indicate a far greater shrinkage than the actual reduction in volume.

5. An embankmeut of very soft material will sometimes "mush" or widen at the sides, with a consequent settling of the top, due to this cause alone.

This subject has called forth much discussion in the technical press and literature. Quotations can be made of figures covering a large range of values, but space will only permit the statement of the conclusions which may be drawn from the large mass of testimony which has been presented.

1. *Volume of loose material.* When material of any character is excavated and deposited loosely in a pile, its volume is always largely in excess of the volume of the excavation. Solid rock will occupy from 60% to 80% *more* space when broken up than when solid. A soft earth will have an excess volume of about 20% to 25%.

2. *Effect of method of depositing.* When material is deposited loosely, as from a trestle, the excess of volume when the embankment is just completed is very large. The time required for final settlement is also very great. When an embankment is formed by the wheelbarrow method, the initial expansion is about as great as when the material is merely dumped from cars. When the material is deposited in small increments from wagons and each layer is subjected to compression from horses' hoofs and from wheels, the contraction during construction is far greater and the additional shrinkage is comparatively small. Wheeled scrapers and drag scrapers will produce even more initial compression.

3. *Time required for final settlement.* This depends partly on the method of formation and also on the character of the

Digitized by Google

material. When a soft loamy soil is deposited loosely, the drying out of the soil during the first long dry season will develop large cracks. Subsequent rains will close these cracks by a general contraction of the whole mass. When the embankment is loosely formed it may take two years before additional settlement becomes inappreciable, but when the method of deposition ensures compression during construction the *subsequent* shrinkage is less in time as well as amount.

4. *Classification of soils with respect to shrinkage.* Loose vegetable surface soil will expand very greatly when excavated and first deposited, but will subsequently shrink to considerably less than its original volume. Clay soils are next in order and the sandy and gravelly soils come at the other end of the list of earthy materials. Rock expands very greatly when first broken up and deposited and there is no appreciable subsequent shrinkage.

97 **Proper allowance for shrinkage.** Specifications for the Mississippi River levees require that there shall be a 10% shrinkage allowance for embankments formed by team work and 25% allowance for wheelbarrow work. It is contended

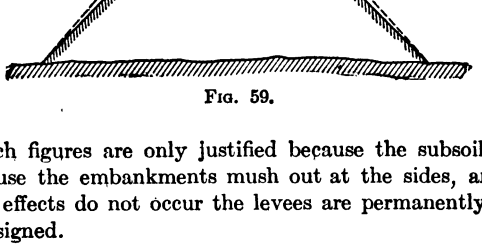


FIG. 59.

that such figures are only justified because the subsoil settles or because the embankments mush out at the sides, and that if these effects do not occur the levees are permanently higher than designed.

It is usual to require that embankments shall be constructed higher than their desired ultimate, as shown in Fig. 59. Since the base does not contract, the contraction may be said to be all vertical. Since a high embankment will unquestionably shrink a greater total amount than a low embankment (whatever the percentage), it follows that an embankment having

Digitized by Google

variable heights (as usual) should have an initial grade-line somewhat like the dotted line *adc* in Fig. 60. Although some such method is essential if there is to be no ultimate sag below the desired grade-line, the policy is sharply criticized. The grade *ad*, even though temporary, may prove objectionable from an operating standpoint. Frequently the allowance is made too great or the shrinkage is not as much as anticipated, and it becomes necessary to cut off the top of the bank. On the other hand, the expense of raising the track after the road is in operation and the inevitable loss of ballast is so great that the danger of being required to fill up a sag should be avoided if possible.

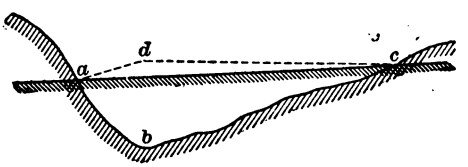


FIG. 60.

A sharp and clear distinction should be made between the *coefficient of extra height* of an embankment and the *coefficient of shrinkage* which determines how many cubic yards of settled embankment may be made from a definite volume of earth or rock measured in the excavation. The values quoted above for the Mississippi levees (from 10% to 25%) refers usually to a very soft soil and includes the effects other than actual contraction of volume. From 8% to 15% is usually quoted as the required extra height of embankments, although it is strenuously claimed by many that 3% or 2% is sufficient, or even that *no* allowance should be made.

The coefficients to determine the amount of settled embankment which may be made from a given volume of earth or rock measured in the excavation, are necessarily subject to variation on account of the method employed and the amount of compression and settlement which will take place during the progress of the work. The following figures have the weight of considerable authority but, if in error, the coefficients are probably high rather than low:

Digitized by Google

Gravel or sand	about	8%
Clay	"	10%
Loam	"	12%
Loose vegetable surface soil	"	15%

It may be noticed from the above table that the harder and cleaner the material the less is the contraction. Perfectly clean gravel or sand would not probably change volume appreciably. The above coefficients of shrinkage and expansion may be used to form the following convenient table:

Material.	To make 1000 cubic yards of embankment will require	1000 cubic yards measured in excavation will make
Gravel or sand	1087 cubic yards	920 cubic yards
Clay	1111 " "	900 " "
Loam	1136 " "	880 " "
Loose vegetable soil	1176 " "	850 " "
Rock, large pieces	714 " "	1400 " "
" small "	625 " "	1600 " "
	measured in excavation	of embankment.

Since writing the above the following values have been adopted by the American Railway Engineering and Maintenance of Way Association as representing standard practice:

COEFFICIENTS OF SHRINKAGE ALLOWANCE FOR DEPOSITING EARTHWORK.

	Trestle filling.	Raising under traffic.
Black dirt	15%	5%
Clay	10%	5%
Sand	6%	5%

98. Methods of forming embankments. Embankments of moderate height are sometimes formed by scraping material with drag scrapers from ditches at the sides, especially if there is little or no cutting to be done in the immediate vicinity. Over a low level swampy stretch this method has the double advantage of building an embankment which is well above the general level and also provides generous drainage ditches which keep the embankment dry. Wheeled scrapers may be used economically up to a distance of 400 feet to excavate

Digitized by Google

cuts and deposit the material on low embankments. Such methods have the advantage of compacting the embankments during construction and reducing future shrinkage.

When carts are used, an embankment of any height may be formed by "dumping over the end" and building to the full height (or even higher to allow for shrinkage) as the embankment proceeds. The method is especially applicable when the material comes from a place as high as or higher than the grade-line, so that no up-hill hauling is necessary. Only a small contractor's plant is required for all of these methods.

Trestles capable of carrying carts, or even cars and locomotives, from which excavated material may be dropped, are found to be economical in spite of the fact that their cost is a construction expense. There is the disadvantage that such embankments require a long time to settle, but there are the advantages that the earth may be hauled by the train load from a distance of perhaps several miles, dumped from the

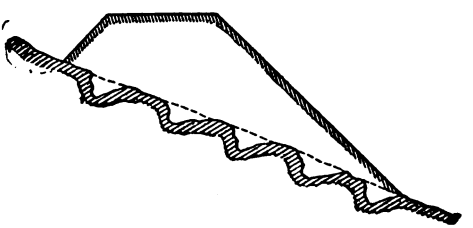


FIG. 61.

cars by train ploughs, or automatically dumped when the material is carried in patent dumping-cars, and all at a comparatively small cost per cubic yard. The disadvantages of slow settlement may be obviated, although at some additional cost, by making the trestle sufficiently strong to support regular traffic until the settlement is complete.

During recent years cableways have been utilized to fill comparatively narrow but deep ravines from material obtainable on either side of the ravine. This method obviates the construction of an excessively high trestle which might otherwise be considered necessary.

When an embankment is to be placed on a steep side hill which has a slippery clay surface, the embankment will some-

Digitized by Google

times slide down the hill, unless means are taken to prevent it. Some sort of bond between the old surface and the new material becomes necessary. This has sometimes been provided by cutting out steps somewhat as is illustrated in Fig. 61. It is possible that a deep ploughing of the surface would accomplish the result just as effectively and much cheaper. The tendency to slip is generally due not only to the nature of the soil but also to the usual accompanying characteristic that the soil is wet and springy. The sub-surface drainage of such a place with tile drains will still further prevent such slipping, which often proves very troublesome and costly.

COMPUTATION OF HAUL.

99. Nature of subject. As will be shown later when analyzing the cost of earthwork, the most variable item of cost is that depending on the distance hauled. As it is manifestly impracticable to calculate the exact distance to which every individual cartload of earth has been moved, it becomes necessary to devise a means which will give at least an equivalent of the haulage of all the earth moved. Evidently the *average* haul for any mass of earth moved is equal to the distance from the center of gravity of the excavation to the center of gravity of the embankment formed by the excavated material. As a rough approximation the center of gravity of a cut (or fill) may sometimes be considered to coincide with the center of gravity of that part of the profile representing it, but the error is frequently very large. The center of gravity may be determined by various methods, but the method of the "mass diagram" accomplishes the same ultimate purpose (the determination of the haul) with all-sufficient accuracy and also furnishes other valuable information.

100. Mass diagram. In Fig. 62 let $A'B' \ldots G'$ represent a profile and grade line drawn to the usual scales. Assume A' to be a point past which no earthwork will be hauled. Such a point is determined by natural conditions, as, for example, a river crossing, or one end of a long level stretch along which no grading is to be done except the formation of a low embankment from the material excavated from ample drainage ditches on each side. Above the profile draw an indefinite horizontal line (ACn in Fig. 62), which may be called the "zero line." Above every station point in the profile draw an ordinate (above or be-

Digitized by Google

low the zero line) which will represent the algebraic sum of the cubic yards of cut and fill (calling cut + and fill −) from the point A' to the point considered. The computations of these ordinates should first be made in tabular form as shown below. In doing this shrinkage must be allowed for by considering how much embankment would actually be made by so many cubic yards of excavation of such material. For example, it will be found that 1000 cubic yards of sand or gravel, measured in place (see § 97), will make about 920 cubic yards of embankment; therefore all cuttings in sand or gravel should be discounted in about this proportion. Excavations in rock should be increased in the proper ratio. In short, all excavations should be valued according to the amount of *settled* embankment that could be made from them. Place in the first column a list of the stations; in the second column, the number of cubic yards of cut or fill between each station and the preceding station; in the third and fourth columns, the kind of material and the proper shrinkage factor; in the fifth column, a repetition of the quantities in cubic yards, except that the excavations are diminished (or increased, in the case of rock) to the number of cubic yards of settled embankment which may be made from them. In the sixth column place the *algebraic sum* of the quantities in the fifth column (calling cuts + and fills −) from the starting-point to the station considered. These algebraic sums at each station will be the ordinates, drawn to some scale, of the mass curve. The scale to be used will depend somewhat on whether

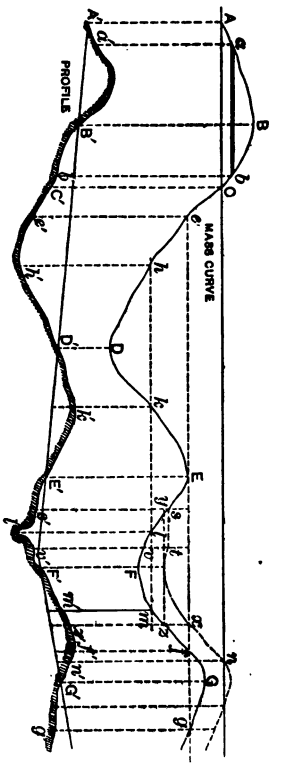


FIG. 62.—MASS DIAGRAM.

Digitized by Google

the work is heavy or light, but for ordinary cases a scale of 5000 cubic yards per inch may be used. Drawing these ordinates to scale, a curve $A, B, \ldots G$ may be obtained by joining the extremities of the ordinates.

Sta.	Yards { cut + fill −	Material.	Shrinkage factor.	Yards, reduced for shrinkage.	Ordinate in mass curve.
46 + 70	.....	.....	.....	.....	0
47	+ 195	Clayey soil	− 10 per cent	+ 175	+ 175
48	+ 1792	" "	− 10 "	+ 1613	+ 1788
+ 60	+ 614	" "	− 10 "	+ 553	+ 2341
49	− 143	.....	.....	− 143	+ 2198
50	− 906	.....	.....	− 906	+ 1292
51	− 1985	.....	.....	− 1985	− 693
52	− 1721	.....	.....	− 1721	− 2414
+ 30	− 112	.....	.....	− 112	− 2526
53	+ 177	Hard rock	+ 60 per cent	+ 283	− 2243
+ 70	+ 180	" "	+ 60 "	+ 289	− 1954
54	− 52	.....	.....	− 52	− 2006
+ 42	− 71	.....	.....	− 71	− 2077
55	+ 276	Clayey soil	− 10 per cent	+ 249	− 1828
56	+ 1242	" "	− 10 "	+ 1118	− 710
57	+ 1302	" "	− 10 "	+ 1172	+ 462

101. Properties of the mass curve.

1. The curve will be rising while over cuts and falling while over fills.

2. A tangent to the curve will be horizontal (as at B, D, E, F, and G) when passing from cut to fill or from fill to cut.

3. When the curve is *below* the "zero line" it shows that material must be drawn *backward* (to the left); and *vice versa*, when the curve is *above* the zero line it shows that material must be drawn *forward* (to the right).

4. When the curve crosses the zero line (as at A and C) it shows (in this instance) that the cut between A' and B' will just provide the material required for the fill between B' and C', and that no material should be hauled past C', or, in general, past any intersection of the mass curve and the zero line.

5. If any horizontal line be drawn (as ab), it indicates that the cut and fill between a' and b' will just balance.

6. When the center of gravity of a given volume of material is to be moved a given distance, it makes no difference (at least theoretically) how far each individual load may be hauled or how any individual load may be disposed of. The summation

Digitized by Google

of the products of each load times the distance hauled will be a constant, whatever the method, and will equal the total volume times the movement of the center of gravity. The *average haul*, which is the movement of the center of gravity, will therefore equal the summation of these products divided by the total volume. If we draw two horizontal parallel lines at an infinitesimal distance dx apart, as at ab, the small increment of cut dx at a' will fill the corresponding increment of fill at b', and this material must be hauled the distance ab. Therefore the product of ab and dx, which is the product of distance times volume, is represented by the area of the infinitesimal rectangle at ab, and the total area ABC represents the summation of volume times distance for all the earth movement between A' and C'. This summation of products divided by the total volume gives the average haul.

7. The horizontal line, tangent at E and cutting the curve at e, f, and g, shows that the cut and fill between e' and E' will just balance, and that a *possible* method of hauling (whether desirable or not) would be to "borrow" earth for the fill between C' and e', use the material between D' and E' for the fill between e' and D', and similarly balance cut and fill between E' and f' and also between f' and g'.

8. Similarly the horizontal line $hklm$ may be drawn cutting the curve, which will show another *possible* method of hauling. According to this plan, the fill between C' and h' would be made by borrowing; the cut and fill between h' and k' would balance; also that between k' and l' and between l' and m'. Since the area $ehDkE$ represents the measure of haul for the earth between e' and E', and the other areas measure the corresponding hauls similarly, it is evident that the sum of the areas $ehDkE$ and $ElFmf$, which is the measure of haul of all the material between e' and f', is largely in excess of the sum of the areas hDk, kEl, and lFm, plus the somewhat uncertain measures of haul due to borrowing material for $e'h'$ and wasting the material between m' and f'. Therefore to make the measure of haul a minimum a line should be drawn which will make the sum of the areas between it and the mass curve a minimum. Of course this is not necessarily the cheapest plan, as it implies more or less borrowing and wasting of material, which *may* cost more than the amount saved in haul. The comparison of the two methods is quite simple, however. Since the amount

Digitized by Google

of fill between e' and h' is represented by the *difference* of the ordinates at e and h, and similarly for m' and f', it follows that the amount to be borrowed between e' and h' will exactly equal the amount wasted between m' and f'. By the first of the above methods the haul is excessive, but is definitely known from the mass diagram, and all of the material is utilized; by the second method the haul is reduced to about one-half, but there is a known quantity in cubic yards wasted at one place and the same quantity borrowed at another. The length of haul necessary for the borrowed material would need to be ascertained; also the haul necessary to waste the other material at a place where it would be unobjectionable. Frequently this is best done by widening an embankment beyond its necessary width. The computation of the relative cost of the above methods will be discussed later (§ 116).

9. Suppose that it were deemed best, after drawing the mass curve, to introduce a trestle between s' and v', thus saving an amount in fill equal to tv. If such had been the original design, the mass curve would have been a straight horizontal line between s and t and would continue as a curve which would be at all points a distance tv above the curve $vFmzfGg$. If the line Ef is to be used as a zero line, its intersection with the new curve at x will show that the material between E' and z' will just balance if the trestle is used, and that the amount of haul will be measured by the area between the line Ex and the broken line $Estx$. The same computed result may be obtained without drawing the auxiliary curve $txn \ldots$ by drawing the horizontal line zy at a distance $xz(=tv)$ below Ex. The amount of the haul can then be obtained by adding the triangular area between Es and the horizontal line Ex, the rectangle between st and Ex, and the irregular area between vFz and $y \ldots z$ (which last is evidently equal to the area between tx and $E \ldots x$). The disposal of the material at the right of z' would then be governed by the indications of the profile and mass diagram which would be found at the right of g'. In fact it is difficult to decide with the best of judgment as to the proper disposal of material without having a mass diagram extending to a considerable distance each side of that part of the road under immediate consideration.

102. **Area of the mass curve.** The area may be computed most readily by means of a planimeter, which is capable with reasonable care of measuring such areas with as great accuracy

Digitized by Google

as is necessary for this work. If no such instrument is obtainable, the area may be obtained by an application of "Simpson's rule." The ordinates will usually be spaced 100 feet apart. Select an *even* number of such spaces, leaving, if necessary, one or more triangles or trapezoids at the ends for separate and independent computation. Let $y_0 \ldots y_n$ be the ordinates, i.e., the number of cubic yards at each station of the mass curve, or the figures of "column six" referred to in § 100. Let the uniform distance between ordinates (=100 feet) be called 1, i.e., one *station*. Then the units of the resulting area will be cubic yards hauled one station. Then the

$$\text{Area} = \tfrac{1}{3}[y_0 + 4(y_1 + y_3 + \ldots y_{(n-1)}) + 2(y_2 + y_4 + \ldots y_{(n-2)}) + y_n]. \quad (70)$$

When an ordinate occurs at a substation, the best plan is to ignore it at first and calculate the area as above. Then, if the difference involved is too great to be neglected, calculate the area of the triangle having the extremity of the ordinate at the substation as an apex, and the extremities of the ordinates at the adjacent stations as the ends of the base. This may be done by finding the ordinate at the substation that would be a proportional between the ordinates at the adjacent full stations. Subtract this from the real ordinate (or *vice versa*) and multiply the difference by $\frac{1}{2} \times 1$. An inspection will often show that the correction thus obtained would be too small to be worthy of consideration. If there is more than one substation between two full stations, the corrective area will consist of two triangles and one or more trapezoids which may be similarly computed, if necessary.

When the zero line (Fig. 62) is shifted to eE, the drop from AC (produced) to E is known in the same units, cubic yards. This constant may be subtracted from the numbers ("column 6," § 100) representing the ordinates, and will thus give, without any scaling from the diagram, the exact value of the modified ordinates.

103. Value of the mass diagram. The great value of the mass diagram lies in the readiness with which different plans for the disposal of material may be examined and compared. When the mass curve is once drawn, it will generally require only a shifting of the horizontal line to show the disposal of the material by any proposed method. The mass diagram also shows the

Digitized by Google

extreme length of haul that will be required by any proposed method of disposal of material. This brings into consideration the "limit of profitable haul," which will be fully discussed in § 116. For the present it may be said that with each method of carrying material there is some limit beyond which the expense of hauling will exceed the loss resulting from borrowing and wasting. With wheelbarrows and scrapers the limit of profitable haul is comparatively short, with carts and tram-cars it is much longer, while with locomotives and cars it may be several miles. If, in Fig. 62, *eE* or *Ef* exceeds the limit of profitable haul, it shows at once that some such line as *hklm* should be drawn and the material disposed of accordingly.

104. Changing the grade line. The formation of the mass curve and the resulting plans as to the disposal of material are based on the mutual relations of the grade line and the surface profile and the amounts of cut and fill which are thereby implied. If the grade line is altered, every cross-section is altered, the amount of cut and fill is altered, and the mass curve is also changed. At the farther limit of the actual change of the grade line the revised mass curve will have (in general) a different ordinate from the previous ordinate at that point. From that point on, the revised mass curve will be parallel to its former position, and the revised curve may be treated similarly to the case previously mentioned in which a trestle was introduced. Since it involves tedious calculations to determine accurately how much the volume of earthwork is altered by a change in grade line, especially through irregular country, the effect on the mass curve of a change in the grade line cannot therefore be readily determined except in an approximate way. Raising the grade line will evidently increase the fills and diminish the cuts, and *vice versa*. Therefore if the mass curve indicated, for example, either an excessively long haul or the necessity for borrowing material (implying a fill) and wasting material farther on (implying a cut), it would be *possible* to diminish the fill (and hence the amount of material to be borrowed) by lowering the grade line near that place, and diminish the cut (and hence the amount of material to be wasted) by raising the grade line at or near the place farther on. Whether the advantage thus gained would compensate for the possibly injurious effect of these changes on the grade line would require patient investigation. But the method outlined shows how the mass

Digitized by Google

curve might be used to indicate a possible change in grade line which might be demonstrated to be profitable.

105. Limit of free haul. It is sometimes specified in contracts for earthwork that *all* material shall be entitled to free haul up to some specified limit, say 500 or 1000 feet, and that all material drawn farther than that shall be entitled to an allowance on the *excess* of distance. It is manifestly impracticable to measure the excess for each load, as much so as to measure the actual haul of each load. The mass diagram also solves this problem very readily. Let Fig. 63 represent a pro-

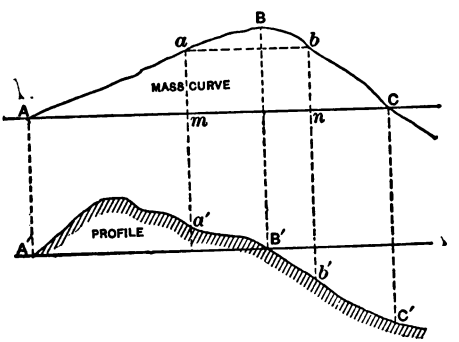


FIG. 63.

file and mass diagram of about 2000 feet of road, and suppose that 800 feet is taken as the limit of free haul. Find two points, a and b, in the mass curve *which are on the same horizontal line* and which are 800 feet apart. Project these points down to a' and b'. Then the cut and fill between a' and b' will just balance, and the cut between A' and a' will be needed for the fill between b' and C'. In the mass curve, the area between the horizontal line ab and the curve aBb represents the haulage of the material between a' and b', which is all free. The rectangle $abmn$ represents the haulage of the material in the cut $A'a'$ across the 800 feet from a' to b'. This is also free. The sum of the two areas Aam and bnC represents the haulage entitled to an allowance, since it is the summation of the products of cubic yards times the *excess* of distance hauled.

If the amount of cut and fill was symmetrical about the point

Digitized by Google

B', the mass curve would be a symmetrical curve about the vertical line through B, and the two limiting lines of free haul would be placed symmetrically about B and B'. In general there is no such symmetry, and frequently the difference is considerable The area $aBbnm$ will be materially changed according as the two vertical lines am and bn, always 800 feet apart, are shifted to the right or left. It is easy to show that the area $aBbnm$ is a *maximum* when ab is horizontal. The minimum value would be obtained either when m reached A or n reached C, depending on the exact form of the curve. Since the position for the minimum value is manifestly unfair, the best definite value obtainable is the maximum, which must be obtained as above described. Since $aBbnm$ is made maximum, the remainder of the area, which is the allowance for overhaul, becomes a minimum. The areas Aam and bCn may be obtained as in § 102. If the whole area $AaBbCA$ has been previously computed, it may be more convenient to compute the area $aBbnm$ and subtract it from the total area.

Since the intersections of the mass curve and the "zero line" mark limits past which no material is drawn, it follows that there will be no allowance for overhaul except where the distance between consecutive intersections of the zero line and mass curve exceeds the limit of free haul.

Frequently all allowances for overhaul are disregarded; the profiles, estimates of quantities, and the required disposal of material are shown to bidding contractors, and they must then make their own allowances and bid accordingly. This method has the advantage of avoiding possible disputes as to the amount of the overhaul allowance, and is popular with railroad companies on this account. On the other hand the facility with which different plans for the disposal of material may be studied and compared by the mass-curve method facilitates the adoption of the most economical plan, and the elimination of uncertainty will frequently lead to a safe reduction of the bid, and so the method is valuable to both the railroad company and the contractor.

ELEMENTS OF THE COST OF EARTHWORK.

106. Analysis of the total cost into items. The variation in the total cost of excavating earthwork, hauling it a greater or less distance, and forming with it an embankment of definite

Digitized by Google

form or wasting it on a spoil bank, is so great that the only possible method of estimating the cost under certain assumed conditions is to separate the total cost into elementary items. Ellwood Morris was perhaps the first to develop such a method —see *Journal of the Franklin Institute*, September and October, 1841. Trautwine used the same general method with some modifications. The following analysis will follow the same general plan, will quote some of the figures given by Morris and by Trautwine, but will also include facts and figures better adapted to modern conditions. Since every item of cost (except interest on cost of plant and its depreciation) is a direct function of the current price of common labor, all calculations will be based on the simple unit of $1 per day. Then the actual cost may be obtained by multiplying the calculated cost under the given conditions by the current price of day labor. When possible, figures will be quoted giving the cost of all items of work on a loose sandy soil which is the easiest to work and also for the cost of the heaviest soils, such as stiff clay and hard pan. These represent the extremes, excluding rock, which will be treated separately. The cost of intermediate grades may be interpolated between the extreme values according to the judgment of the engineer as to the character of the soil.

The possible division into items varies greatly according to the method adopted, but the differentiation into items given below (which is strictly applicable to the old fashioned simpler methods of work) can usually be applied to any other method by merely combining or eliminating some of the items. The items are

1. Loosening the natural soil.
2. Loading the soil into whatever carrier may be used.
3. Hauling excavated material from excavation to embankment or spoil bank.
4. Spreading or distributing the soil on the embankment.
5. Keeping roadways or tracks in good running order.
6. Trimming cuts to their proper cross-section (sometimes called "sandpapering").
7. Repairs, wear, depreciation, and interest on cost of plant.
8. Superintendence and incidentals.

107. Item 1. Loosening. (a) **Ploughs.** Very light sandy soils can frequently be shovelled without any previous loosening, but it is generally economical, even with very light material,

Digitized by Google

to use a plough. Morris quotes, as the results of experiments, that a three-horse plough would loosen from 250 to 800 cubic yards of earth per day, which at a valuation of $5 per day would make the cost per yard vary from 2 cents to 0.6 cent. Trautwine estimates the cost on the basis of two men handling a two-horse plough at a total cost of $3.87 per day, being $1 each for the men, 75 c. for each horse, and an allowance of 37 c. for the plough, harness, etc. From 200 to 600 cubic yards is estimated as a fair day's work, which makes a cost of 1.9 c. to 0.65 c. per yard, which is substantially the same estimate as above. Extremely heavy soils have sometimes been loosened by means of special ploughs operated by traction-engines.

Gillette estimates that "a two-horse team with a driver and a man holding the plough will loosen 25 cubic yards of fairly tough clay, or 35 cubic yards of gravel and loam per hour." For ten hours per day this would be 250 to 350 cubic yards per day. These values are neither as high nor as low as the extremes above noted. It is probably very seldom that a soil will be so light that a two-horse (or three-horse) plough can loosen as much as 600 (or 800) cubic yards per day.

It is sometimes necessary to plough up a macadamized street. This may be done by using as a plough a pointed steel bar which is fastened to a very strong plough frame. A preliminary hole must be made which will start the bar under the macadam shell. Then, as the plough is drawn ahead, the shell is ripped up. Four or six horses, or even a traction-engine, are used for such work. Gillette quotes two such cases where the cost of such loosening was 2 c. and 6 c. per cubic yard, with common labor at 15 c. per hour. Two-thirds of such figures will reduce them to the $1 per day basis. The cost for ploughing *on the* $1 *per day basis* may therefore be summarized as follows:

For very loose sandy soils.....	0.6 c.	per cubic yard
" " heavy clay "	2.0 c.	" " "
" hard pan and macadam, up to ...	4.0 c.	" " "

(b) **Picks.** When picks are used for loosening the earth, as is frequently necessary and as is often done when ploughing would perhaps be really cheaper, an estimate* for a fair day's

* Trautwine.

Digitized by Google

work is from 14 to 60 cubic yards, the 14 yards being the estimate for stiff clay or cemented gravel, and the 60 yards the estimate for the lightest soil that would require loosening. At $1 per day this means about 7 c. to 1.7 c. per cubic yard, which is about three times the cost of ploughing. Five feet of the face is estimated * as the least width along the face of a bank that should be allowed to enable each laborer to work with freedom and hence economically.

(c) **Blasting.** Although some of the softer shaly rocks may be loosened with a pick for about 15 to 20 c. per yard, yet rock in general, frozen earth, and sometimes even compact clay are most economically loosened by blasting. The subject of blasting will be taken up later, §§ 117–123.

(d) **Steam-shovels.** The items of loosening and loading merge together with this method, which will therefore be treated in the next section.

108. **Item 2. Loading.** (a) **Hand-shovelling.** Much depends on proper management, so that the shovellers need not wait unduly either for material or carts. With the best of management considerable time is thus lost, and yet the intervals of rest need not be considered as entirely lost, as it enables the men to work, while actually loading, at a rate which it would be physically impossible for them to maintain for ten hours. Seven shovellers are sometimes allowed for each cart; otherwise there should be five, two on each side and one in the rear. Economy requires that the number of loads per cart per day should be made as large as possible, and it is therefore wise to employ as many shovellers as can work without mutual interference and without wasting time in waiting for material or carts. The figures obtainable for the cost of this item are unsatisfactory on account of their large disagreements. The following are quoted as the number of cubic yards that can be loaded into a cart by an average laborer in a working day of ten hours, the lower estimate referring to heavy soils, and the higher to light sandy soils: 10 to 14 cubic yards (Morris), 12 to 17 cubic yards (Haskoll), 18 to 22 cubic yards (Hurst), 17 to 24 cubic yards (Trautwine), 16 to 48 cubic yards (Ancelin). As these estimates are generally claimed to be based on actual experience, the discrepancies are probably due to differences of management. If the

* Hurst.

Digitized by Google

average of 15 to 25 cubic yards be accepted, it means, on the basis of $1 per day, 6.7 c. to 4 c. per cubic yard. These estimates apply only to earth. *Rockwork* costs more, not only because it is harder to handle, but because a cubic yard of solid rock, measured in place, occupies about 1.8 cubic yards when broken up, while a cubic yard of earth will occupy about 1.2 cubic yards. Rockwork will therefore require about 50% more loads to haul a given volume, *measured in place*, than will the same nominal volume of earthwork. The above authorities give estimates for loading rock varying from 6.9 c. to 10 c. per cubic yard. The above estimates apply only to the loading of carts or cars with shovels or by hand (loading masses of rock). The cost of loading wheelbarrows and the cost of scraper work will be treated under the item of hauling.

(b) **Steam-shovels.*** Whenever the magnitude of the work will warrant it there is great economy in the use of steam-shovels. These have a "bucket" or "dipper" on the end of a long beam, the bucket having a capacity varying from ½ to 2½ cubic yards. Steam-shovels handle all kinds of material from the softest earth to shale rock, earthy material containing large boulders, tree-stumps, etc. The record of work done varies from 200 to 1000 cubic yards in 10 hours. They perform all the work of loosening and loading. Their economical working requires that the material shall be hauled away as fast as it can be loaded, which usually means that cars on a track, hauled by horses or mules, or still better by a locomotive, shall be used. The expenses for a steam-shovel, costing about $5000, will average about $1000 per month. Of this the engineer may get $100; the fireman $50; the cranesman $90; repairs perhaps $250 to $300; coal, from 15 to 25 tons, cost very variable on account of expensive hauling; water, a very uncertain amount, sometimes costing $100 per month; about five laborers and a foreman, the laborers getting $1.25 per day and the foreman $2.50 per day, which will amount to $227.50 per month. This gang of laborers is employed in shifting the shovel when necessary, taking up and relaying

* For a thorough treatment of the capabilities, cost, and management of steam-shovels the reader is referred to "Steam-shovels and Steam-shovel Work," by E. A. Hermann. D. Van Nostrand Co., New York.
This book is now out of print. "Earthwork and its Cost," by H. P. Gillette, to which the student is referred for a more elaborate exposition of the subject, has used many of Hermann's cuts.

Digitized by Google

tracks for the cars, shifting loaded and unloaded cars, etc. In shovelling through a deep cut, the shovel is operated so as to undermine the upper parts of the cut, which then fall down within reach of the shovel, thus increasing the amount of material handled for each new position of the shovel. If the material is too tough to fall down by its own weight, it is sometimes found economical to employ a gang of men to loosen it or even blast it rather than shift the shovel so frequently. Non-condensing engines of 50 horse-power use so much water that the cost of water-supply becomes a serious matter if water is not readily obtainable. The lack of water facilities will often justify the construction of a pipe line from some distant source and the installation of a steam-pump. Hence the seemingly large estimate of $100 per month for water-supply, although under favorable circumstances the cost may almost vanish. The larger steam-shovels will consume nearly a ton of coal per day of 10 hours. The expense of hauling this coal from the nearest railroad or canal to the location of the cut is often a very serious item of expense and may easily double the cost per ton. Some steam-shovels have been constructed to be operated by electricity obtained from a plant perhaps several miles away. Such a method is especially advantageous when fuel and water are difficult to obtain.

The following general requirements and specifications were recommended in 1907 by the American Railway Engineering and Maintenance of Way Association:

Three important cardinal points should be given careful attention in the selection of a steam-shovel. These are in their order

(1) Care in the selection, inspection and acceptance of all material that enters into every part of the machine.

(2) Design for strength.

(3) Design for production.

GENERAL SPECIFICATIONS.

Weight of shovel: Seventy (70) tons.

Capacity of dipper: Two and one-half (2½) yards.

Steam pressure: One hundred and twenty (120) pounds.

Clear height above rail of shovel track at which dipper should unload: Sixteen (16) feet.

Digitized by Google

Depth below rail of shovel track at which dipper should dig Four (4) feet.

Number of movements of dipper per minute from time of entering bank to entering bank: Three (3).

Character of hoist: Cable.

Character of swing: Cable.

Character of housing: Permanent for all employes.

Capacity of tank: Two thousand (2000) gallons.

Capacity of coal-bunker: Four (4) tons.

Spread of jack arm: Eighteen (18) feet. A special short arm should be provided.

Form of steam-shovel track: "T" rails on ties.

Length of rails for ordinary work: Six (6) feet.

Form of rail joint: Strap.

Manufacturers of steam-shovels will sometimes "guarantee" that certain of their shovels will excavate, say 3000 cubic yards of earth per day of ten hours. Even if it were possible for a shovel to fill a car at the rate of 5 cubic yards per minute, it is always impracticable to maintain such a speed, since a shovel must always wait for the shifting of cars and for the frequent shifting of the shovel itself. There are also delays due to adjustments and minor breakdowns. The best shovel records are made when the cars are large—other things being equal. The item of interest and depreciation of the plant is very large in steam-shovel work. This will be discussed further later. The cost of loading alone will usually come to between 3 and 4 c. per cubic yard. The cost of shifting the cars so as to place them successively under the shovel, haul them to the dumping place, dump them and haul them back, will generally be as much more. Gillette quotes five jobs on one railroad where the total cost for loading and hauling varied from 5.9 c. to 11.4 c. per cubic yard. But as these figures are based on *car* measurement, the cost per cubic yard in *place* measurement must be increased about one-fourth, or from 7.4 c. to 14.2 c.

109. Item 3. Hauling. The cost of hauling depends on the number of round trips per day that can be made by each vehicle employed. As the cost of each vehicle is practically the same whether it makes many trips or few, it becomes important that the number of trips should be made a maximum, and to that end there should be as little delay as possible in loading and

Digitized by Google

unloading. Therefore devices for facilitating the passage of the vehicles have a real money value.

(a) **Carts.** The average speed of a horse hauling a two-wheeled cart has been found to be 200 feet per minute, a little slower when hauling a load and a little faster when returning empty. This figure has been repeatedly verified. It means an allowance of one minute for each 100 feet (or "station") of "lead—the lead being the distance the earth is hauled." The time lost in loading, dumping, waiting to load, etc., has been found to average 4 minutes per load. Representing the number of stations (100 feet) of lead by s, the number of loads handled in 10 hours (600 minutes) would be $600 \div (s+4)$. The number of loads per cubic yard, measured in the bank, is differentiated by Morris into three classes, viz.:

3 loads per cubic yard in descending hauling;
3½ " " " " " level hauling; and
4 " " " " " ascending hauling.

Attempts have been made to estimate the effect of the grade of the roadway by a theoretical consideration of its rate, and of the comparative strength of a horse on a level and on various grades. While such computations are always practicable on a railway (even on a temporary construction track), the traction on a temporary earth roadway is always very large and so very variable that any refinements are useless. On railroad earthwork the hauling is generally nearly level or it is *descending*—forming embankments on low ground with material from cuts in high ground. The only common exception occurs when an embankment is formed from borrow-pits on low ground. One method of allowing for ascending grade is to add to the horizontal distance 14 times the difference of elevation for work with carts and 24 times the difference of elevation for work with wheelbarrows, and use that as the lead. For example, using carts, if the lead is 300 feet and there is a difference of elevation of 20 feet, the lead would be considered equivalent to $300 + (14 \times 20) = 580$ feet on a level.

Trautwine assumes the average load for all classes of work to be ½ cubic yard, which figure is justified by large experience. Using one figure for all classes of work simplifies the calculations and gives the number of cubic yards carried per day of 10 hours equal to $\frac{600}{3(s+4)}$. Dividing the cost of a cart per day by the

Digitized by Google

number of cubic yards carried gives the cost of hauling per yard. In computing the cost of a cart per day, Trautwine refers to the practice of having one driver manage four carts, thus making a charge of 25 c. per day for each cart for the driver. Although this might be an economical method when the haul is very long, it is not economical for short hauls. A safer estimate is to allow not more than two carts per driver and in many cases a driver for each cart. Some contractors employ a driver for each cart and then require that the drivers shall assist in loading. The policy to be adopted is sometimes dependent on labor union conditions, which may demand that drivers must not assist in loading. The supply of labor and the amount of work on hand have a great influence on the methods of work which a contractor may adopt, for a strike will often disarrange all plans.

The cost of a horse and cart must practically include a charge for the time of the horse on Sundays, rainy days and holidays. The cost of repairs of cart and harness is generally included in this item for simplicity, but, under a strict application of the analysis suggested in § 106, it should properly be included under Item 7, Repairs, etc.

Since the time required for loading loose rock is greater than for earthwork, less loads will be hauled per day. The time allowance for loading, etc., is estimated by Trautwine as 6 minutes instead of 4 as for earth. Considering the great expansion of rock when broken up (see § 97), one cubic yard of solid rock, measured in place, would furnish the equivalent of five loads of earthwork of $\frac{4}{3}$ cubic yard. Therefore, on the basis of five loads per cubic yard, the number of cubic yards handled per day per cart would be $\frac{600}{5(s+6)}$.

Let C represent the daily cost of a horse and cart and of the proportional cost of the driver (according to the number of carts handled by one driver), then the cost per cubic yard, *measured in the cut*, for hauling may be given by the formula:

$$\left.\begin{aligned} &\text{Cost per cu. yd. of hauling earth in carts} = \frac{C \times 3(s+4)}{600} \\ &\text{" " " " " " rock " " } = \frac{C \times 5(s+6)}{600} \end{aligned}\right\} \quad . \quad (71)$$

Digitized by Google

(b) **Wagons.** For longer leads (i.e., from ½ to ¾ of a mile) wagons drawn by two horses have been found most economical. The wagons have bottoms of loose thick narrow boards and are unloaded very easily and quickly by lifting the individual boards and breaking up the continuity of the bottom, thus depositing the load directly underneath the wagon. The capacity is about one cubic yard. The cost may be estimated on the same principle as that for carts.

The number of wagon trips per 10 hours will depend somewhat on the management of the shovellers. Too many shovellers per wagon is not economical, measured in yards shovelled per man, although it may reduce the time consumed in loading any one wagon. At an average figure of 20 cubic yards, measured in place, per shoveller per 10 hours, seven shovellers would load 14 cubic yards per hour or one cubic yard in 4.3 minutes. This would be the allowance for a wagon with a capacity of about 1½ yards of loose earth. Adding time for unloading, waiting to load and other possible "lost time," there is probably a total of six minutes. This figure will vary very considerably according to the number of shovellers per wagon, the capacity of the wagon, the type of wagon (whether self-dumping) and other details in the method of management. Adopting six minutes as the time used for loading, unloading, and other "lost time," the formula becomes.

$$\text{Cost per cubic yard of hauling in wagons} = \frac{C(s+6)}{600\,c}, \quad \ldots \quad (71a)$$

in which C is the cost of the wagon, team and driver per day of 10 hours; s is the distance hauled in stations of 100 feet, and c is the capacity of the wagon in cubic yards, *place measurement*, which should be about three fourths of the nominal capacity of the wagon for earth and about sixty per cent when handling rock.

(c) **Wheelbarrows.** Gillette has computed from observations that a man will trundle a wheelbarrow at the rate of 250 feet per minute or 1.25 *stations* of lead per minute for the round trip. The time required for loading is estimated at 2¼ minutes and for unloading, adjusting wheeling planks, short rests, etc., ¾ minute, or a total of three minutes per trip for all purposes except hauling. Gillette allows for a load only 1/15 cubic yard,

Digitized by Google

measured in place, or about 1/11 yard, 2.5 cubic feet, on the wheelbarrow. With notation as before

$$\text{Cost per cubic yard of loading and hauling earth in wheelbarrows} = \frac{C \times 15(1.25s+3)}{600}. \quad (72)$$

In this equation C is the cost of both loading and hauling, and usually includes the allowance (Item 7) for the cost, repairs and depreciation of the wheelbarrows, whose service is very short lived. Trautwine estimates this at five cents per day or a total of $1.05 for labor and wheelbarrow.

The number of wheelbarrow loads required for a cubic yard of rock, measured in place, is about twenty-four. The time required for loading should also be increased about one fourth; the time required for all purposes except hauling is therefore about 3.75 minutes, and the corresponding equation becomes

$$\text{Cost per cubic yard of loading and hauling rock in wheelbarrows} = \frac{C \times 24(1.25s+3.75)}{600}. \quad (72a)$$

(d) **Scrapers.** These are made in three general ways, "buck" scrapers, "drag" scrapers and "wheeled" scrapers. The buck scraper in its original form consisted merely of a wide plank, shod with an iron strap on the lower edge and provided with a pole and a small platform on which the driver may stand to weight it down. The earth is not loaded on to any receptacle and carried, but is merely pushed over the ground. Notwithstanding the apparent inefficiency of the method, its extreme simplicity has caused its occasional adoption for the construction of canal embankments out of material from the bed of the canal. The occasions are rare when their use for railroad work would be practicable, and even then drag scrapers would probably be preferable.

A drag scraper is an immense "scoop shovel" about three feet long and three feet wide. There are usually two handles and a bail in front by which it is dragged by a team of horses. The nominal capacity varies from 7.5 cubic feet for the largest sizes, down to 3 cubic feet for the "one-horse" size, but these figures must be discounted by perhaps 40 or 50% for the actual average volume (as measured in the cut) loaded on during one scoop. The expansion of the earth during loosening is alone respons-

Digitized by Google

ible for a discount of 25%. These scrapers cost from $10 to $18.

A wheeled scraper is essentially an extra-large drag scraper which may be raised by a lever and carried on a pair of large wheels. Their nominal capacity ranges from 10 to 17 cubic feet, which should usually be liberally discounted when estimating output. They are loaded by dropping the scoop so that it scrapes up its load. The lever raises the scoop so that the load is carried on wheels instead of being dragged. At the dump the scoop is tipped so as to unload it. The movement of the scraper is practically continuous. They cost from $40 to $75. Their advantages over drag scrapers consist (1) in their greater capacity, (2) in the economy of transporting the load on wheels instead of by dragging, and (3) in the far greater length of haul over which the earth may be economically handled.

Morris estimated the speed of drag scrapers to be 140 feet per minute, or 70 feet of *lead* per minute. The "lead" should be here interpreted as the average distance from the center of the pit to the center of the dump. Gillette declares the speed to be 220 feet per minute. Some of this variation may be due to differences in the method of measuring the distance actually travelled, especially when the lead is very short, since the scraper teams must always travel a considerable extra distance at each end in order to turn around most easily. This extra distance is practically constant whether the lead is long or short. Gillette quotes an instance where the length of lead was actually about 20 feet, but the scraper teams travelled about 150 feet for each load carried. On this account Gillette adopts a minimum of 75 feet of lead no matter how short the lead actually may be. Of course the speed depends considerably on how strictly the men are kept to their work and also on the care which may be taken to obtain a full load for each scraper. As a compromise between Morris's and Gillette's estimates we may adopt the convenient rate of speed of 200 feet per minute, or 100 feet of lead per minute. There should also be allowed for the time lost in loading and unloading and for travelling the extra distance travelled by the teams in making the circuit, $1\frac{1}{2}$ minutes. Allowing the average value of seven loads per cubic yard and letting C represent the cost of scraper team and driver per day, we have for the cost as follows:

Digitized by Google

$$\left.\begin{array}{l}\text{Cost per cubic yard of loading and}\\ \text{hauling earth in drag scrapers}\end{array}\right\} = \frac{C \times 7(s+1\frac{1}{3})}{600}. \quad . \quad . \quad (73)$$

In this formula C should include the cost of not only the driver, team, and scraper, but also the proper proportion of the wages of an extra man, who assists each driver in loading his scraper, and whose wages should be divided among the two (or three) scrapers to which he is assigned. Scraper work nearly always implies ploughing, the cost of which should be computed as under Item 1.

When a low embankment is formed from borrow-pits on each side of the road, it may be done with scrapers, which move from one borrow-pit to the other, taking a load alternately from each side to the center and making but one half turn for each load carried. This reduces the time lost in turning by one third of a minute and reduces the constant in the numerator in Eq. (73) from $1\frac{1}{3}$ to 1. In this case the lead will usually be not greater than 75 feet, and therefore, if we consider this as a minimum value, s will ordinarily equal .75 and the quantity in the parenthesis will equal 1.75.

When using wheeled scrapers the catalogue capacity, which varies from 9 or 10 feet for a No. 1 scraper to 16 or 17 feet for a No. 3 scraper, must be reduced to 5 loads per cubic yard (place measurement) for a No. 1 scraper and to $2\frac{1}{2}$ loads per cubic yard for a No. 3, not only on account of the expansion of the earth during loosening, but also on account of the impracticability of loading these scrapers to their maximum nominal capacity. When the haul or lead for wheeled scrapers is 300 feet or over, it will be justifiable to employ shovellers to fill up the bowl of the shovel, especially when the soil is tough and when it is impracticable to fill the shovel even approximately full by the ordinary method. A snatch team to assist in loading the scrapers it also economical, especially with the larger scrapers. The proportionate number of snatch teams to the total number of scrapers of course depends on the length of haul. The cost of these extra shovellers and extra snatch teams must be divided proportionally among the number of scrapers assisted, in determining the value C in the formula given below. The extra time to be allowed on account of turning, loading, and dumping is about $1\frac{1}{2}$ minutes. The speed is considered one station of lead per minute as before. If we call C the average

Digitized by Google

daily cost of one scraper and n the capacity of the scraper, or the number of loads per cubic yard, we may write the following formula:

$$\left.\begin{array}{l}\text{Cost per cubic yard of loading and}\\ \text{hauling earth in wheeled scrapers}\end{array}\right\} = \frac{C \times n(s+1\frac{1}{2})}{600}. \quad . \quad . \quad (73a)$$

(e) Cars and horses. The items of cost by this method are (*a*) charge for horses employed, (*b*) charge for men employed strictly in hauling, (*c*) charge for shifting rails when necessary, (*d*) repairs, depreciation, and interest on cost of cars and track. Part of this cost should strictly be classified under items 5 and 7, mentioned in § 106, but it is perhaps more convenient to estimate them as follows:

The traction of a car on rails is so very small that grade resistance constitutes a very large part of the total resistance if the grade is 1% or more. For all ordinary grades it is sufficiently accurate to say that the grade resistance is to the gross weight as the rise is to the distance. If the distance is supposed to be measured along the slope, the proportion is strictly true; i.e., on a 1% grade the grade resistance is 1 lb. per 100 of weight or 20 lbs. per ton. If the resistance on a level at the usual velocity is $\frac{1}{120}$, a grade of 1:120 (0.83%) will exactly double it. If the material is hauled *down* a grade of 1:120, the cars will run by gravity after being started. The work of hauling will then consist practically of hauling the empty cars up the grade. The grade resistance depends only on the rate of grade and the weight, but the tractive resistance will be *greater per ton of weight* for the unloaded than for the loaded cars. The tractive power of a horse is less on a grade than on a level, not only because the horse raises his own weight in addition to the load, but is anatomically less capable of pulling on a grade than on a level. In general it will be possible to plan the work so that loaded cars need not be hauled *up* a grade, unless an embankment is to be formed from a low borrow-pit, in which case another method would probably be advisable. These computations are chiefly utilized in designing the method of work—the proportion of horses to cars. An example may be quoted from English practice (Hurst), in which the cars had a capacity of $3\frac{1}{2}$ cubic yards, weighing 30 cwt. empty. Two horses took five "wagons" $\frac{3}{4}$ of a mile on a level

Digitized by Google

railroad and made 15 journeys per day of 10 hours, i.e., they handled 250 yards per day. In addition to those on the "straight road," another horse was employed to make up the train of loaded wagons. With a short lead the straight-road horses were employed for this purpose. In the above example the number of men required to handle these cars, shift the tracks, etc., is not given, and so the exact cost of the above work cannot be analyzed. It may be noticed that the two horses travelled 22½ miles per day, drawing in one direction a load, including the weight of the cars, of about 57,300 lbs., or 28.65 net tons. Allowing $\frac{1}{120}$ as the necessary tractive force, it would require a pull of 477.5 lbs., or 239 lbs. for each horse. With a velocity of 220 feet per minute this would amount to 1½ horse-power per horse, exerted for only a short time, however, and allowing considerable time for rest and for drawing only the empty cars. Gillette claims that the rolling resistance for such cars on a contractor's track should be considered as 40 lbs. per ton (the equivalent of a 2% grade) and quotes many figures to support the assertion. Unquestionably the resistance on tracks with very light rails, light ties with wide spacing and no tamping, would be very great and might readily amount to 40 lbs. per ton. In the above case, the resistance could not have been much if any over $\frac{1}{120}$. A resistance of 40 lbs. per ton would have required each horse to pull about 573 lbs. for nearly five hours per day, beside pulling the empty cars the rest of the time. This is far greater exertion than any ordinary horse can maintain. The cars generally used in this country have a capacity of 1½ cubic yards and cost about $65 apiece. Besides the shovellers and dumping-gang, several men and a foreman will be required to keep the track in order and to make the constant shifts that are necessary. Two trains are generally used, one of which is loaded while the other is run to the dump. Some passing-place is necessary, but this is generally provided by having a switch at the cut and running the trains on each track alternately. This insures a train of cars always at the cut to keep the shovellers employed. The cost of hauling per cubic yard can only be computed when the number of laborers, cars, and horses employed are known, and these will depend on the lead, on the character of the excavation, on the grade, if any, etc., and must be so proportioned that the shovellers need not wait for cars to fill, nor the dumping-gang

Digitized by Google

for material to handle, nor the horses and drivers for cars to haul. Much skill is necessary to keep a large force in smooth running order.

(f) **Cars and locomotives.** 30-lb. rails are the lightest that should be used for this work, and 35- or 40-lb. rails are better. One or two narrow-gauge locomotives (depending on the length of haul), costing about $2500 each, will be necessary to handle two trains of about 15 cars each, the cars having a capacity of about 2 cubic yards and costing about $100 each. Some cars can be obtained as low as $70. A force of about five men and a foreman will be required to shift the tracks. The track-shifters, except the foreman, may be common laborers. The dumping-gang will require about seven men. Even when the material is all taken *down* grade the grades may be too steep for the safe hauling of loaded cars down the grade, or for hauling empty cars up the grade. Under such circumstances temporary trestles are necessary to reduce the grade. When these are used, the uprights and bracing are left in the embankment—only the stringers being removed. This is largely a necessity, but is partially compensated by the fact that the trestle forms a core to the embankment which prevents lateral shifting during settlement. The average speed of the trains may be taken as 10 miles per hour or 5 miles of lead per hour. The time lost in loading and unloading is estimated (Trautwine) as 9 minutes or .15 of an hour. The number of trips per day of 10 hours will equal $\frac{10}{\frac{1}{5}(\text{miles of lead}) + .15}$ or $\frac{50}{(\text{miles of lead}) + .75}$. Of course this quotient *must* be a whole number. Knowing the number of trains and their capacity, the total number of cubic yards handled is known, which, divided into the total daily cost of the trains, will give the cost of hauling per yard. The daily cost of a train will include

(*a*) Wages of engineer, who frequently fires his own engine;

(*b*) Fuel, about ¼ to 1 ton of bituminous coal, depending on work done;

(*c*) Water, a very variable item, frequently costing $3 to $5 per day;

(*d*) Repairs, variable, frequently at rate of 50 to 60% per year;

(*e*) Interest on cost and depreciation, 16 to 40%.

To these must be added, to obtain the total cost of haul,

(*f*) Wages of the gang employed in shifting track.

Digitized by Google

The above calculation for the number of train loads depends on the assumption that 9 minutes is total time lost by a locomotive for each round trip. If the haul is very short it may readily happen that a steam-shovel cannot fill one train of cars before the locomotive has returned with a load of empties and is ready to haul a loaded train away. The estimation of the number of train loads is chiefly useful in planning the work so as to have every tool working at its highest efficiency. Usually the capacity of the steam-shovel or the ability to promptly "spot" the cars under the shovel is the real limiting agent which determines the daily output.

110. **Choice of method of haul dependent on distance.** In light side-hill work in which material need not be moved more than 12 or 15 feet, i.e., moved *laterally* across the roadbed, the earth may be moved most cheaply by mere shovelling. Beyond 12 feet scrapers are more economical. At about 100 feet drag-scrapers and wheelbarrows are equally economical. Between 100 and 200 feet wheelbarrows are generally cheaper than either carts or drag-scrapers, but wheeled scrapers are always cheaper than wheelbarrows. Beyond 500 feet two-wheeled carts become the most economical up to about 1700 feet; then four-wheeled wagons become more economical up to 3500 feet. Beyond this cars on rails, drawn by horses or by locomotives, become cheaper. The economy of cars on rails becomes evident for distances as small as 300 feet provided the volume of the excavation will justify the outlay. Locomotives will always be cheaper than horses and mules, providing the work to be done is of sufficient magnitude to justify the purchase of the necessary plant and risk the loss in selling the plant ultimately as second-hand equipment, or keeping the plant on hand and idle for an indefinite period waiting for other work. Horses will not be economical for distances much over a mile. For greater distances locomotives are more economical, but the question of "limit of profitable haul" (§ 116) must be closely studied, as the circumstances are certainly not common when it is advisable to haul material much over a mile.

111. Item 4. SPREADING. The cost of spreading varies with the method employed in dumping the load. When the earth is

Digitized by Google

tipped over the edge of an embankment there is little if any necessary work. Trautwine allows about ¼ c. per cubic yard for keeping the dumping-places clear and in order. This would represent the wages of one man at $1 per day attending to the unloading of 1200 two-wheeled carts each carrying ½ cubic yard. 1200 carts in 10 hours would mean an average of two per minute, which implies more rapid and efficient work than may be depended on. The allowance is probably too small. When the material is dumped in layers some levelling is required, for which Trautwine allows 50 to 100 cubic yards as a fair day's work, costing from 1 to 2 cents per cubic yard. The cost of spreading will not ordinarily exceed this and is frequently nothing—all depending on the method of unloading. It should be noted that Mr. Morris's examples and computations (Jour. Franklin Inst., Sept. 1841) disregard altogether any special charge for this item.

112. Item 5. KEEPING ROADWAYS IN ORDER. This feature is important as a measure of true economy, whatever the system of transportation, but it is often neglected. A petty saving in such matters will cost many times as much in increased labor in hauling and loss of time. With some methods of haul the cost is best combined with that of other items.

(a) Wheelbarrows. Wheelbarrows should generally be run on planks laid on the ground. The adjusting and shifting of these planks is done by the wheelers, and the time for it is allowed for in the "¾ minute for short rests, adjusting the wheeling plank, etc." The actual cost of the planks must be added, but it would evidently be a very small addition per cubic yard in a large contract. When the wheelbarrows are run on planks placed on "horses" or on trestles the cost is very appreciable; but the method is frequently used with great economy. The variations in the requirements render any general estimate of such cost impracticable.

(b) Carts and wagons. The cost of keeping roadways in order for carts and wagons is sometimes estimated merely as so much per cubic yard, but it is evidently a function of the *lead*. The work consists in draining off puddles, filling up ruts, picking up loose stones that may have fallen off the loads, and in general doing everything that will reduce the traction as much as possible. Temporary inclines, built to avoid excessive grade

Digitized by Google

at some one point, are often measures of true economy. Trautwine suggests $\frac{1}{10}$ c. per cubic yard per 100 feet of lead for earthwork and $\frac{2}{10}$ c. for rockwork, as an estimate for this item when carts are used.

(c) **Cars.** When cars are used a shifting-gang, consisting of a foreman and several men (say five), are constantly employed in shifting the track so that the material may be loaded and unloaded where it is desired. The average cost of this item may be estimated by dividing the total daily cost of this gang by the number of cubic yards handled in one day.

112a. **Item 6. Trimming cuts to their proper cross-section.** This process, often called "sand-papering," must be treated as an expense, since the payment received for the very few cubic yards of earth excavated is wholly inadequate to pay for the work involved. Gillette quotes bids of 2 cents per *square* yard of surface trimmed, and from this argues that, for *average* excavations, it adds to the cost *four* cents per cubic yard of the total excavation. The shallower the cut the greater is the proportionate cost. Of course the actual cost to the contractor will depend largely on the accuracy of outline demanded by the engineer or inspector.

113. **Item 7. Repairs, wear, depreciation, and interest on cost of plant.** The amount of this item evidently depends upon the character of the soil—the harder the soil the worse the wear and depreciation. The *interest on cost* depends on the current borrowing value of money. The estimate for this item has already been included in the allowances for horses, carts, ploughs, harness, wheelbarrows, steam-shovels, etc. Trautwine estimates $\frac{1}{4}$ c. per cubic yard for picks and shovels. Depreciation is generally a large percentage of the cost of earth-working tools, the life of all being limited to a few years, and of many tools to a few months.

114. **Item 8. Superintendence and incidentals.** The incidentals include the cost of water-boys, timekeepers, watchmen, blacksmiths, fences, and other precautions to protect the public from possible injury, cost of casualty insurance for workmen, etc. Although the cost of some of these sub-items may be definitely estimated, others are so uncertain that it is only possible to make a lump estimate and add say 5 to 7% of the sum of the previous items for this item.

Digitized by Google

115. Contractor's profit and contingencies. The word "contingencies" here refers to the abnormal expenses caused by freshets, continued wet weather, and "hard luck," as distinguished from mere incidentals which are really normal expenses. They are the expenses which literally cannot be foreseen, and on which the contractor must "take chances." They are therefore included with the expected profit. The allowance for these two elements combined is variously estimated up to 25% of the previously estimated cost of the work, according to the sharpness of the competition, the contractor's confidence in the accuracy of his estimates, and the possible uncertainty as to true cost owing to unfavorable circumstances. The contractor's real profit may vary considerably from this. He often pays clerks, boards and lodges the laborers in shanties built for the purpose, or keeps a supply-store, and has various other items both of profit and expense. His profit is largely dependent on skill in so handling the men that all can work effectively without interference or delays in waiting for others. An unusual season of bad weather will often affect the cost very seriously. It is a common occurrence to find that two contractors may be working on the same kind of material and under precisely similar conditions and at the same price, and yet one may be making money and the other losing it—all on account of difference of management.

116. Limit of profitable haul. As intimated in §§ 103 and 110, there is with every method of haul a limit of distance beyond which the expense for excessive hauling will exceed the loss resulting from borrowing and wasting. This distance is somewhat dependent on local conditions, thus requiring an independent solution for each particular case, but the general principles involved will be about as follows: Assume that it has been determined, as in Fig. 62, that the cut and fill will exactly balance between two points, as between e and x, assuming that, as indicated in § 101 (9), a trestle has been introduced between s and t, thus altering the mass curve to $Estxn\ldots$ Since there is a balance between A' and C', the material for the fill between C' and e' must be obtained either by "borrowing" in the immediate neighborhood or by transportation from the excavation

Digitized by Google

between z' and n'. If cut and fill have been approximately balanced in the selection of grade line, as is ordinarily done, borrowing material for the fill $C'e'$ implies a wastage of material at the cut $z'n'$. To compare the two methods, we may place against the plan of borrowing and wasting, (*a*) cost, if any, of extra right of way that may be needed from which to obtain earth for the fill $C'e'$; (*b*) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the borrow-pit and of the fill, and the other expenses incidental to borrowing M cubic yards for the fill $C'e'$; (*c*) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the cut $z'n'$ and of the spoil-bank, and the other expenses incidental to wasting M cubic yards at the cut $z'n'$. (*d*) cost, if any, of land needed for the spoil-bank. The cost of the other plan will be the cost of loosening, loading, hauling (the hauling being represented by the trapezoidal figure *Cexn*), and the other expenses incidental to making the fill $C'e'$ with the material from the cut $z'n'$, the amount of material being M cubic yards, which is represented in the figure by the vertical ordinate from e to the line *Cn*. The difference between these costs will be the cost, if any, of land for borrow-pit and spoil-bank *plus* the cost of loosening, loading, etc. (except hauling and roadways) of M cubic yards, *minus* the difference in cost of the excessive haul from *Ce* to *xn* and the comparatively short hauls from borrow-pit and to spoil-bank.

As an illustration, taking some of the estimates previously given for operating with average material, the cost of all items, except hauling and roadways, would be about as follows: loosening, with plough, 1.2 c., loading 5.0 c., spreading 1.5 c., wear, depreciation, etc., .25 c., superintendence, etc., 1.5 c.; total 8.95 c. Suppose that the haul for both borrowing and wasting averages 100 feet or 1 station. Then the cost of haul per yard, using carts, would be (§ 109, *a*) $[125\times3(1+4)]\div600 = 3.125$ c. The cost of roadways would be about 0.1 c. per yard, making a total of 3.225 c. per cubic yard. Assume $M = 10000$ cubic yards and the area $Cexn = 180000$ yards-stations or the equivalent of 10000 yards hauled 1800 feet. This haul would cost $[125\times3(18+4)]\div600 = 13.75$ c. per cubic yard. The cost of roadways will be $18\times.1$ or 1.8 c., making a total of 15,55 c. for hauling and roadways. The difference of cost of hauling and roadways will be $15.55-(2\times3.225) = 9.10$ c. per yard or $910

Digitized by Google

for the 10000 yards. Offsetting this is the cost of loosening, etc., 10000 yards, at 8.95 c., costing $895. These figures may be better compared as follows:

Long Haul.	Loosening, etc., 10000 yards, @ 8.95 c.	$ 895.
	Hauling, " 10000 " @ 15.55 c.	1555.
		$2450.
Borrowing and Wasting.	Loosening, etc., 10000 yards (borrowed), @ 8.95 c.	$895.
	" " 10000 " (wasted), @ 8.95 c.	895.
	Hauling, etc., 10000 " (borrowed), @ 3.225 c.	322.50
	" " 10000 " (wasted), @ 3.225 c.	322.50
		$2435.00

These costs are practically balanced, but no allowance has been made for right of way. If any considerable amount had to be paid for that, it would decide this particular case in favor of the long haul. This shows that *under these conditions* 1800 feet is *about* the limit of profitable haul, the land costing nothing extra.

BLASTING.

117. Explosives. The effect of blasting is due to the extremely rapid expansion of a gas which is developed by the decomposition of a very small amount of solid matter. Blasting compounds may be divided into two general classes, (*a*) slow-burning and (*b*) detonating. Gunpowder is a type of the slow-burning compounds. These are generally ignited by heat; the ignition proceeds from grain to grain; the heat and pressure produced are comparatively low. Nitro-glycerine is a type of the detonating compounds. They are exploded by a shock which *instantaneously* explodes the whole mass. The heat and pressure developed are far in excess of that produced by the explosion of powder. Nitro-glycerine is so easily exploded that it is very dangerous to handle. It was discovered that if the nitro-glycerine was absorbed by a spongy material like infusorial earth, it was much less liable to explode, while its power when actually exploded was practically equal to that of the amount of pure nitro-glycerine contained in the dynamite, which is the name given to the mixture of nitro-glycerine and infusorial earth. Nitro-glycerine is expensive; many other explosive chemical compounds which properly belong to the slow-burning

Digitized by Google

class are comparatively cheap. It has been conclusively demonstrated that a mixture of nitro-glycerine and some of the cheaper chemicals has a greater explosive force than the sum of the strengths of the component parts when exploded separately. Whatever the reason, the fact seems established. The reason is possibly that the explosion of the nitro-glycerine is sufficiently powerful to produce a *detonation* of the other chemicals, which is impossible to produce by ordinary means, and that this explosion caused by detonation is more powerful than an ordinary explosion. The majority of the explosive compounds and "powders" on the market are of this character—a mixture of 20 to 60 per cent. of nitro-glycerine with variable proportions of one or more of a great variety of explosive chemicals.

The choice of the explosive depends on the character of the rock. A hard brittle rock is most effectively blasted by a detonating compound. The rapidity with which the full force of the explosive is developed has a shattering effect on a brittle substance. On the contrary, some of the softer tougher rocks and indurated clays are but little affected by dynamite. The result is but little more than an enlargement of the blast-hole. Quarrying must generally be done with blasting-powder, as the quicker explosives are too shattering. Although the results obtained by various experimenters are very variable, it may be said that pure nitro-glycerine is eight times as powerful as black powder, dynamite (75% nitro-glycerine) six times, and gun-cotton four to six times as powerful. For open work where time is not particularly valuable, black powder is by far the cheapest, but in tunnel-headings, whose progress determines the progress of the whole work, dynamite is so much more effective and so expedites the work that its use becomes economical.

118. Drilling. Although many very complicated forms of drill-bars have been devised, the best form (with slight modifications to suit circumstances) is as shown in Fig. 64, (*a*) and (*b*). The width should flare at the bottom (*a*) about 15 to 30%. For hard rock the curve of the edge should be somewhat flatter and for soft rock somewhat more curved than shown, Fig. 64, (*a*). Sometimes the angle of the two faces is varied from that given, Fig. 64, (*b*), and occasionally the edge is purposely blunted so as to give a crushing rather than a cutting effect. The drills will require sharpening for each 6 to 18 inches depth of hole, and will require a new edge to be worked every 2 to 4 days.

Digitized by Google

For drilling vertical holes the *churn-drill* is the most economical. The drill-bar is of iron, about 6 to 8 feet long, 1¼″ in diameter, weighs about 25 to 30 lbs., and is shod with a piece of steel welded on. The bar is lifted a few inches between each blow, turned partially around, and allowed to fall, the impact doing the work. From 5 to 15 feet of holes, depending on the character of the rock, is a fair day's work—10 hours. In very soft rocks even more than this may be done. This method is

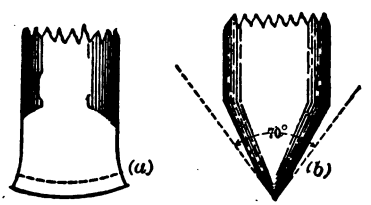


FIG. 64.

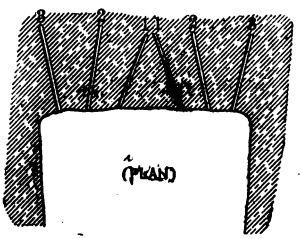
inapplicable for inclined holes or even for vertical holes in confined places, such as tunnel-headings. For such places the only practical *hand* method is to use hammers. This may be done by light drills and light hammers (one-man work), or by heavier drills held by one man and struck by one or two men with heavy hammers. The conclusion of an exhaustive investigation as to the relative economy of light or heavy hammers is that the light-hammer method is more economical for the softer rocks, the heavy-hammer method is more economical for the harder rocks, but that the light-hammer method is always more expeditious and hence to be preferred when time is important.

The subject of machine rock-drills is too vast to be treated here. The method is only practicable when the amount of work to be done is large, and especially when time is valuable. The machines are generally operated by compressed air for tunnel-work, thus doing the additional service of supplying fresh air to the tunnel-headings where it is most needed. The cost per foot of hole drilled is quite variable, but is usually somewhat less than that of hand-drilling—sometimes but a small fraction of it.

119. **Position and direction of drill-holes.** As the cost of drilling holes is the largest single item in the total cost of blasting, it is necessary that skill and judgment should be used in so

Digitized by Google

locating the holes that the blasts will be most effective. The greatest effect of a blast will evidently be in the direction of the "line of least resistance." In a strictly homogeneous material this will be the shortest line from the center of the explosive to the surface. The variations in homogeneity on account of laminations and seams require that each case shall be judged according to experience. In open-pit blasting it is generally easy to obtain two and sometimes three exposed faces to the rock, making it a simple matter to drill holes so that a blast will do effective work. When a solid face of rock must be broken into, as in a tunnel-heading, the work is necessarily ineffectual and expensive. A conical or wedge-shaped mass will first be blown out by simultaneous blasts in the holes marked 1, Fig. 65; blasts in the holes marked 2 and 3 will then complete the cross-section of the heading. A great saving in cost may often be secured by skilfully taking advantage of seams, breaks, and irregularities. When the work is economically done there is but little noise or throwing of rock, a covering of old timbers and branches of trees generally sufficing to confine the smaller pieces which would otherwise fly up.



DRILL HOLES IN TUNNEL HEADING

FIG. 65.

120. Amount of explosive. The amount of explosive required varies as the cube of the line of least resistance. The best results are obtained when the line of least resistance is $\frac{3}{4}$ of the depth of the hole; also when the powder fills about $\frac{1}{3}$ of the hole. For average rock the amount of powder required is as follows:

Line of least resistance.....	2 ft.	4 ft.	6 ft.	8 ft.
Weight of powder.....	$\frac{1}{4}$ lb.	2 lbs.	$6\frac{3}{4}$ lbs.	16 lbs.

Strict compliance with all of the above conditions would require that the diameter of the hole should vary for every case. While this is impracticable, there should evidently be some variation in the size of the hole, depending on the work to be done. For example, a 1″ hole, drilled 2′ 8″ deep, with its line of least resistance 2′. and loaded with $\frac{1}{4}$ lb. of powder, would

Digitized by Google

be filled to a depth of 9½", which is nearly ¼ of the depth. A 3" hole, drilled 8' deep, with its line of least resistance 6', and loaded with 6¾ lbs. of powder, would be filled to a depth of over 28", which is also nearly ¼ of the depth. One pound of blasting-powder will occupy about 28 cubic inches. Quarrying necessitates the use of numerous and sometimes repeated light charges of powder, as a heavy blast or a powerful explosive like dynamite is apt to shatter the rock. This requires more powder to the cubic yard than blasting for mere excavation, which may usually be done by the use of ¼ to ½ lb. of powder per cubic yard of easy open blasting. On account of the great resistance offered by rock when blasted in headings in tunnels, the powder used per cubic yard will run up to 2, 4, and even 6 lbs. per cubic yard. As before stated, nitro-glycerine is about eight times (and dynamite about six times) as powerful as the same *weight* of powder.

121. **Tamping.** Blasting-powder and the slow-burning explosives require thorough tamping. Clay is probably the best, but sand and fine powdered rock are also used. Wooden plugs, inverted expansive cones, etc., are periodically reinvented by enthusiastic inventors, only to be discarded for the simpler methods. Owing to the extreme rapidity of the development of the force of a nitro-glycerine or dynamite explosion, tamping is not so essential with these explosives, although it unquestionably adds to their effectiveness. Blasting under water has been effectively accomplished by merely pouring nitro-glycerine into the drilled holes through a tube and then exploding the charge without any tamping except that furnished by the superincumbent water. It has been found that air-spaces about a charge make a material reduction in the effectiveness of the explosion. It is therefore necessary to carefully ram the explosive into a solid mass. Of course the liquid nitro-glycerine needs no ramming, but dynamite should be rammed with a *wooden* rammer. Iron should be carefully avoided in ramming gunpowder. A copper bar is generally used.

122. **Exploding the charge.** Black powder is generally exploded by means of a fuse which is essentially a cord in which there is a thin vein of gunpowder, the cord being protected by tar, extra linings of hemp, cotton, or even gutta-percha. The fuse is inserted into the middle of the charge, and the tamping carefully packed around it so that it will not be injured. To

Digitized by Google

produce the detonation required to explode nitro-glycerine and dynamite, there must be an initial explosion of some easily ignited explosive. This is generally accomplished by means of caps containing fulminating-powder which are exploded by electricity. The electricity (in one class of caps) heats a very fine platinum wire to redness, thereby igniting the sensitive powder, or (in another class) a spark is caused to jump through the powder between the ends of two wires suitably separated. Dynamite can also be exploded by using a small cartridge of gunpowder which is itself exploded by an ordinary fuse.

123. Cost. Trautwine estimates the cost of blasting (for mere excavation) as averaging 45 cents per cubic yard, falling as low as 30 cents for easy but *brittle* rock, and running up to 60 cents and even $1 when the cutting is shallow, the rock especially tough, and the strata unfavorably placed. Soft tough rock frequently requires more powder than harder brittle rock.

124. Classification of excavated material. The classification of excavated material is a fruitful source of dispute between contractors and railroad companies, owing mainly to the fact that the variation between the softest earth and the hardest rock is so gradual that it is very difficult to describe distinctions between different classifications which are unmistakable and indisputable. The classification frequently used is (*a*) earth, (*b*) loose rock, and (*c*) solid rock. As blasting is frequently used to loosen "loose rock" and even "earth" (if it is frozen), the fact that blasting is employed cannot be used as a criterion, especially as this would (if allowed) lead to unnecessary blasting for the sake of classifying material as rock.

Earth. This includes clay, sand, gravel, loam, decomposed rock and slate, boulders or loose stones not greater than 1 cubic foot (3 cubic feet, P. R. R.), and sometimes even "hard-pan." In general it will signify material which *can* be loosened by a plough with two horses, or with which one picker can keep one shoveller busy.

Loose rock. This includes boulders and loose stones of more than one cubic foot and less than one cubic yard; stratified rock, not more than six inches thick, separated by a stratum of clay; also all material (not classified as earth) which may be loosened by pick or bar and which "*can* be quarried without blasting, although blasting may occasionally be resorted to."

Digitized by Google

Solid rock includes all rock found in masses of over one cubic yard which cannot be removed except by blasting.

It is generally specified that the engineer of the railroad company shall be the judge of the classification of the material, but frequently an appeal is taken from his decisions to the courts.

125. Specifications for earthwork. The following specifications, issued by the Norfolk and Western R. R., represent the average requirements. It should be remembered that very strict specifications invariably increase the cost of the work, and frequently add to the cost more than is gained by improved quality of work.

1. The grading will be estimated and paid for by the cubic yard, and will include clearing and grubbing, and all open excavations, channels, and embankments required for the formation of the roadbed, and for turnouts and sidings; cutting all ditches or drains about or contiguous to the road; digging the foundation-pits of all culverts, bridges, or walls; reconstructing turnpikes or common roads in cases where they are destroyed or interfered with; changing the course or channel of streams; and all other excavations or embankments connected with or incident to the construction of said Railroad.

2. All grading, except where otherwise specified, whether for cuts or fills, will be measured in the excavations and will be classified under the following heads, viz.: Solid Rock, Loose Rock, Hard-pan, and Earth.

SOLID ROCK shall include all rock occurring in masses which, in the judgment of the said Engineer Maintenance of Way, may be best removed by blasting.

LOOSE ROCK shall include all kinds of shale, soapstone, and other rock which, in the judgment of the said Engineer Maintenance of Way, can be removed by pick and bar, and is soft and loose enough to be removed without blasting, although blasting may be occasionally resorted to; also, detached stone of less than one (1) cubic yard and more than one (1) cubic foot.

HARD-PAN shall consist of tough indurated clay or cemented gravel, which requires blasting or other equally expensive means for its removal, or which cannot be ploughed with less than four horses and a railroad plough, or which requires two pickers to a shoveller, the said Engineer Maintenance of Way to be the judge of these conditions.

Digitized by Google

EARTH shall include all material of an earthy nature, of whatever name or character, not unquestionably loose rock or hardpan as above defined.

POWDER. The use of powder in cuts will not be considered as a reason for any other classification than earth, unless the material in the cut is clearly other than earth under the above specifications.

3. Earth, gravel, and other materials taken from the excavations, except when otherwise directed by the said Engineer Maintenance of Way or his assistant, shall be deposited in the adjacent embankment; the cost of removing and depositing which, when the distance necessary to be hauled is not more than sixteen hundred (1600) feet, shall be included in the price paid for the excavation.

4. EXTRA HAUL will be estimated and paid for as follows: whenever material from excavations is necessarily hauled a greater distance than sixteen hundred (1600) feet, there shall be paid in addition to the price of excavation the price of extra haul per 100 feet, or part thereof, after the first 1600 feet; the necessary haul to be determined in each case by the said Engineer Maintenance of Way or his assistant, from the profile and cross-sections, and the estimates to be in accordance therewith.

5. All embankments shall be made in layers of such thickness and carried on in such manner as the said Engineer Maintenance of Way or his assistant may prescribe, the stone and heavy materials being placed in slopes and top. And in completing the fills to the proper grade such additional heights and fulness of slope shall be given them, to provide for their settlement, as the said Engineer Maintenance of Way, or his assistant, may direct. Embankments about masonry shall be built at such times and in such manner and of such materials as the said Engineer Maintenance of Way or his assistant may direct.

6. In procuring materials for embankments from without the line of the road, and in wasting materials from cuttings, the place and manner of doing it shall in each case be indicated by the Engineer Maintenance of Way or his assistant; and care must be taken to injure or disfigure the land as little as possible. Borrow-pits and spoil-banks must be left by the Contractor in regular and sightly shape.

7. The lands of the said Railroad Company shall be cleared to the extent required by the said Engineer Maintenance of

Digitized by Google

Way, or his assistant, of all trees, brushes, logs, and other perishable materials, which shall be destroyed by burning or deposited in heaps as the said Engineer Maintenance of Way, or his assistant, may direct. Large trees must be cut not more than two and one-half (2½) feet from the ground, and under embankments less than four (4) feet high they shall be cut close to the ground. All small trees and bushes shall be cut close to the ground.

8. Clearing shall be estimated and paid for by the acre or fraction of an acre.

9. All stumps, roots, logs, and other obstructions shall be grubbed out, and removed from all places where embankments occur less than two (2) feet in height; also, from all places where excavations occur and from such other places as the said Engineer Maintenance of Way or his assistant may direct.

10. Grubbing shall be estimated and paid for by the acre or fraction of an acre.

11. Contractors, when directed by the said Engineer Maintenance of Way or his assistant in charge of the work, will deposit on the side of the road, or at such convenient points as may be designated, any stone, rock, or other materials that they may excavate; and all materials excavated and deposited as above, together with all timber removed from the line of the road, will be considered the property of the Railroad Company, and the Contractors upon the respective sections will be responsible for its safe-keeping until removed by said Railroad Company, or until their work is finished.

12. Contractors will be accountable for the maintenance of safe and convenient places wherever public or private roads are in any way interfered with by them during the progress of the work. They will also be responsible for fences thrown down, and for gates and bars left open, and for all damages occasioned thereby.

13. Temporary bridges and trestles, erected to facilitate the progress of the work, in case of delays at masonry structures from any cause, or for other reasons, will be at the expense of the Contractor.

14. The line of road or the gradients may be changed in any manner, and at any time, if the said Engineer Maintenance of Way or his assistant shall consider such a change necessary or expedient; but no claim for an increase in prices of excavation

Digitized by Google

or embankment on the part of the Contractor will be allowed or considered unless made in writing before the work on that part of the section where the alteration has been made shall have been commenced. The said Engineer Maintenance of Way or his assistant may also, on the conditions last recited, increase or diminish the length of any section for the purpose of more nearly equalizing or balancing the excavations and embankments, or for any other reason.

15. The roadbed will be graded as directed by the said Engineer Maintenance of Way or his assistant, and in conformity with such breadths, depths, and slopes of cutting and filling as he may prescribe from time to time, and no part of the work will be finally accepted until it is properly completed and dressed off at the required grade.

Digitized by Google

CHAPTER IV.

TRESTLES.

126. Extent of use. Trestles constitute from 1 to 3% of the length of the average railroad. It was estimated in 1889 that there was then about 2400 miles of single-track railway trestle in the United States, divided among 150,000 structures and estimated to cost about $75,000,000. The annual charge for maintenance, estimated at ⅛ of the cost, therefore amounted to about $9,500,000 and necessitated the annual use of perhaps 300,000,000 ft. B. M. of timber. The corresponding figures at the present time must be somewhat in excess of this. The magnitude of this use, which is causing the rapid disappearance of forests, has resulted in endeavors to limit the use of timber for this purpose. Trestles may be considered as justifiable under the following conditions:

a. Permanent trestles.

1. Those of *extreme* height—then called viaducts and frequently constructed of iron or steel, as the Kinzua viaduct, 302 ft. high.

2. Those across waterways—*e.g.*, that across Lake Pontchartrain, near New Orleans, 22 miles long.

3. Those across swamps of soft deep mud, or across a river-bottom, liable to occasional overflow.

b. Temporary trestles.

1. To open the road for traffic as quickly as possible—often a reason of great financial importance.

2. To quickly replace a more elaborate structure, destroyed by accident, on a road already in operation, so that the interruption to traffic shall be a minimum.

3. To form an earth embankment with earth brought from a distant point by the train-load, when such a measure would cost less than to borrow earth in the immediate neighborhood.

4. To bridge an opening temporarily and thus allow time to learn the regimen of a stream in order to better proportion the

Digitized by Google

size of the waterway and also to facilitate bringing *suitable* stone for masonry from a distance. In a new country there is always the double danger of either building a culvert too small, requiring expensive reconstruction, perhaps after a disastrous washout, or else wasting money by constructing the culvert unnecessarily large. Much masonry has been built of a very poor quality of stone because it could be conveniently obtained and because good stone was unobtainable except at a prohibitive cost for transportation. Opening the road for traffic by the use of temporary trestles obviates both of these difficulties.

127. Trestles vs. embankments. Low embankments are very much cheaper than low trestles both in first cost and maintenance. Very high embankments are very expensive to construct, but cost comparatively little to maintain. A trestle of equal height may cost much less to construct, but will be expensive to maintain—perhaps $\frac{1}{8}$ of its cost per year. To determine the height beyond which it will be cheaper to maintain a trestle rather than build an embankment, it will be necessary to allow for the cost of maintenance. The height will also depend on the relative cost of timber, labor, and earthwork. At the present average values, it will be found that for less heights than 25 feet the *first cost* of an embankment will *generally* be less than that of a trestle; this implies that a permanent trestle should never be constructed with a height less than 25 feet except for the reasons given in § 126. The height at which a permanent trestle is certainly cheaper than earthwork is more uncertain. A high grade line joining two hills will invariably imply at least a culvert if an embankment is used. If the culvert is built of masonry, the cost of the embankment will be so increased that the height at which a trestle becomes economical will be materially reduced. The cost of an embankment increases much more rapidly than the height—with very high embankments more nearly as the square of the height—while the cost of trestles does not increase as rapidly as the height. Although local circumstances may modify the application of any set rules, it is probably seldom that it will be cheaper to build an embankment 40 or 50 feet high than to permanently maintain a wooden trestle of that height. A steel viaduct would probably be the best solution of such a case. These are frequently used for permanent structures, especially when very high. The cost of maintenance is much less than that of wood, which makes the

Digitized by Google

use of iron or steel preferable for permanent trestles unless wood is abnormally cheap. Neither the cost nor the construction of iron or steel trestles will be considered in this chapter.

128. Two principal types. There are two principal types of wooden trestles—pile trestles and framed trestles. The great objection to pile trestles is the rapid rotting of the portion of the pile which is underground, and the difficulty of renewal. The maximum height of pile trestles is about 30 feet, and even this height is seldom reached. Framed trestles have been constructed to a height of considerably over 100 feet They are frequently built in such a manner that any injured piece may be readily taken out and renewed without interfering with traffic. Trestles consist of two parts—the supports called "bents," and the stringers and floor system. As the stringers and floor system are the same for both pile and framed trestles, the "bents" are all that need be considered separately.

PILE TRESTLES.

129. Pile bents. A pile bent consists generally of four piles driven into the ground deep enough to afford not only sufficient vertical resistance but also lateral resistance. On top of these piles is placed a horizontal "cap." The caps are fastened to the tops of the piles by methods illustrated in Fig. 66. The method of fastening shown in each case should not be considered as applicable only to the particular type of pile bent used to illustrate it. Fig. 66 (*a* and *d*) illustrates a mortise-joint with a hardwood pin about 1¼″ in diameter. The hole for the pin should be bored separately through the cap and the mortise, and the hole through the cap should be at a slightly higher level than that through the mortise, so that the cap will be drawn down tight when the pin is driven. Occasionally an iron dowel (an iron pin about 1½″ in diameter and about 6″ long) is inserted partly in the cap and partly in the pile. The use of drift-bolts, shown in Fig. 66 (*b*), is cheaper in first cost, but renders repairs and renewals very troublesome and expensive. "Split caps," shown in Fig. 66 (*c*), are formed by bolting two half-size strips on each side of a tenon on top of the pile. Repairs are very easily and cheaply made without interference with the traffic and without injuring other pieces of the bent. The smaller pieces are more easily obtainable in a sound condition; the

Digitized by Google

decay of one does not affect the other, and the first cost is but little if any greater than the method of using a single piece. For further discussion, see § 136.

For very light traffic and for a height of about 5 feet three vertical piles will suffice, as shown in Fig. 66 (*a*). Up to a height

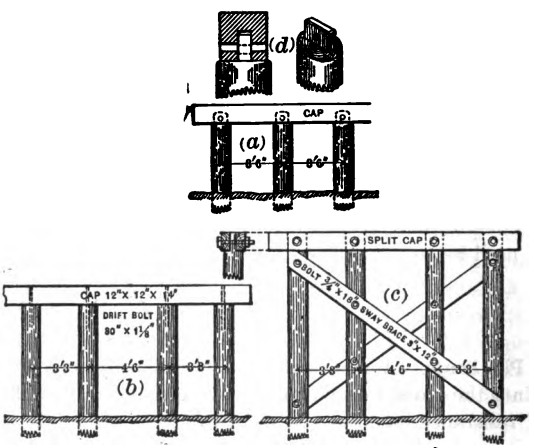


FIG. 66.

of 8 or 10 feet four piles may be used without sway-bracing, as in Fig 66 (*b*), if the piles have a good bearing. For heights greater than 10 feet sway-bracing is generally necessary. The outside piles are frequently driven with a batter varying from 1 : 12 to 1 : 4.

Piles are made, if possible, from timber obtained in the vicinity of the work. Durability is the great requisite rather than strength, for almost any timber is strong enough (except as noted below) and will be suitable if it will resist rapid decay. The following list is quoted as being in the order of preference on account of durability:

1. Red cedar	5. White pine	9. White oak	12. Black oak
2. Red cypress	6. Redwood	10. Post-oak	13. Hemlock
3. Pitch-pine	7. Elm	11. Red oak	14. Tamarac
4. Yellow pine	8. Spruce		

Red-cedar piles are said to have an average life of 27 years with a possible maximum of 50 years, but the timber is rather

Digitized by Google

weak, and if exposed in a river to flowing ice or driftwood is apt to be injured. Under these circumstances oak is preferable, although its life may be only 13 to 18 years.

130. **Methods of driving piles.** The following are the principal methods of driving piles:

a. A hammer weighing 2000 to 3000 lbs. or more, sliding in guides, is drawn up by horse-power or a portable engine, and allowed to fall *freely*.

b. The same as above except that the hammer does not fall freely, but drags the rope and revolving drum as it falls and is thus quite materially retarded. The mechanism is a little more simple, but is less effective, and is sometimes made deliberately deceptive by a contractor by retarding the blow, in order to apparently indicate the requisite resistance on the part of the pile.

The above methods have the advantage that the mechanism is cheap and can be transported into a new country with comparative ease, but the work done is somewhat ineffective and costly compared with some of the more elaborate methods given below.

c. Gunpowder pile-drivers, which automatically explode a cartridge every time the hammer falls. The explosion not only forces the pile down, but throws up the hammer for the next blow. For a given height of fall the effect is therefore doubled. It has been shown by experience, however, that when it is attempted to use such a pile-driver rapidly the mechanism becomes so heated that the cartridges explode prematurely, and the method has therefore been abandoned.

d. Steam pile-drivers, in which the hammer is operated directly by steam. The hammer falls freely a height of about 40 inches and is raised again by steam. The effectiveness is largely due to the rapidity of the blows, which does not allow time between the blows for the ground to settle around the pile and increase the resistance, which does happen when the blows are infrequent. "The hammer-cylinder weighs 5500 lbs., and with 60 to 75 lbs. of steam gives 75 to 80 blows per minute. With 41 blows a large unpointed pile was driven 35 feet into a hard clay bottom in half a minute." Such a driver would cost about $800.

The above four methods are those usual for dry earth. In very soft wet or sandy soils, where an unlimited supply of water

Digitized by Google

is available, the *water-jet* is sometimes employed. A pipe is fastened along the side of the pile and extends to the pile-point. If water is forced through the pipe, it loosens the sand around the point and, rising along the sides, decreases the side resistance so that the pile sinks by its own weight, aided perhaps by extra weights loaded on. This loading may be accomplished by connecting the top of the pile and the pile-driver by a block and tackle so that a portion of the weight of the pile-driver is continually thrown on the pile.

Excessive driving frequently fractures the pile below the surface and thereby greatly weakens its bearing power. To prevent excessive "brooming" of the top of the pile, owing to the action of the hammer, the top should be protected by an iron ring fitted to the top of the pile. The "brooming" not only renders the driving ineffective and hence uneconomical, but vitiates the value of any test of the bearing power of the pile by noting the sinking due to a given weight falling a given distance. If the pile is so soft that brooming is unavoidable, the top should be adzed off frequently, and especially should it be done just before the final blows which are to test its bearing-power.

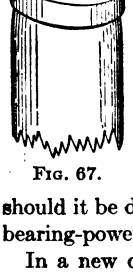


FIG. 67.

In a new country judgment and experience will be required to decide intelligently whether to employ a simple drop-hammer machine, operated by horse-power and easily transported but uneconomical in operation, or a more complicated machine working cheaply and effectively after being transported at greater expense.

131. Pile-driving formulæ. If R = the resistance of a pile, and s the set of the pile during the last blow, w the weight of the pile-hammer, and h the fall during the last blow, then we may state the approximate relation that $Rs = wh$, or $R = \frac{wh}{s}$. This is the basic principle of all rational formulæ, but the maximum weight which a pile will sustain after it has been driven some time is by no means equal to the resistance of the pile during the last blow. There are also many other modifying elements which have been variously allowed for in the many proposed formulæ. The formulæ range from the extreme of empirical simplicity to very complicated attempts to allow

Digitized by Google

properly for all modifying causes. As the simplest rule, specifications sometimes require that the piles shall be driven until the pile will not sink more than 5 inches under five consecutive blows of a 2000-lb. hammer falling 25 feet. The "*Engineering News* formula" * gives the safe load as $\frac{2wh}{s+1}$, in which w = weight of hammer, h = fall in *feet*, s = set of pile in *inches* under the last blow. This formula is derived from the above basic formula by calling the safe load $\frac{1}{4}$ of the final resistance, and by adding (arbitrarily) 1 to the final set (s) as a compensation for the extra resistance caused by the settling of earth around the pile between each blow. This formula is used only for ordinary hammer-driving. When the piles are driven by a steam pile-driver the formula becomes safe load $= \frac{2wh}{s+0.1}$. For the "gunpowder pile-driver," since the explosion of the cartridge drives the pile in with the same force with which it throws the hammer upward, the effect is *twice* that of the fall of the hammer, and the formula becomes safe load $= \frac{4wh}{s+0.1}$. In these last two formulæ the constant in the denominator is changed from $s+1$ to $s+0.1$. The constant (1.0 or 0.1) is supposed to allow, as before stated, for the effect of the extra resistance caused by the earth settling around the pile between each blow. The more rapid the blows the less the opportunity to settle and the less the proper value of the constant.

The above formulæ have been given on account of their simplicity and their practical agreement with experience. Many other formulæ have been proposed, the majority of which are more complicated and attempt to take into account the weight of the pile, resistance of the guides, etc. While these elements, as well as many others, have their influence, their effect is so overshadowed by the indeterminable effect of other elements—as, for example, the effect of the settlement of earth around the pile between blows—that it is useless to attempt to employ anything but a purely empirical formula.

Examples. 1. A pile was driven with an ordinary hammer weighing 2500 pounds until the sinking under five consecutive blows was 15½ inches. The fall of the hammer during the last

* *Engineering News*, Nov. 17, 1892.

Digitized by Google

blows was 24 feet. What was the safe bearing power of the pile?

$$\frac{2wh}{s+1}=\frac{2\times2500\times24}{(\frac{1}{4}\times15.5)+1}=\frac{120000}{4.1}=29300 \text{ pounds.}$$

2. Piles are being driven into a firm soil with a steam pile-driver until they show a *safe* bearing power of 20 tons. The hammer weighs 5500 pounds and its fall is 40 inches. What should be the sinking under the final blow?

$$40000=\frac{2wh}{s+0.1}=\frac{2\times5500\times3.33}{s+0.1},$$

$$s=\frac{36667}{40000}-0.1=.81 \text{ inch.}$$

132. Pile-points and pile-shoes. Piles are generally sharpened to a blunt point. If the pile is liable to strike boulders, sunken logs, or other obstructions which are liable to turn the point, it is necessary to protect the point by some form of shoe. Several forms in cast iron have been used, also a wrought-iron shoe, having four "straps" radiating from the apex, the straps being nailed on to the pile, as shown in Fig. 68 (*b*). The cast-iron form shown in Fig. 68 (*a*) has a base cast around a drift-bolt. The recess on the top of the base receives the bottom of the pile and prevents a tendency to split the bottom of the pile or to force the shoe off laterally.

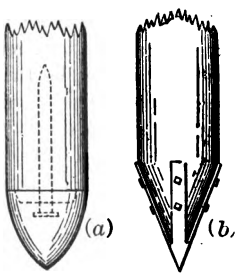


FIG. 68.

133. Details of design. No theoretical calculations of the strength of pile bents need be attempted on account of the extreme complication of the theoretical strains, the uncertainty as to the real strength of the timber used, the variability of that strength with time, and the insignificance of the economy that would be possible even if exact sizes could be computed. The piles are generally required to be not less than 10″ or 12″ in diameter at the large end. The P. R. R. requires that they shall

Digitized by Google

be "not less than 14 and 7 inches in diameter at butt and small end respectively, exclusive of bark, which must be removed." The removal of the bark is generally required in good work. Soft *durable* woods, such as are mentioned in § 129, are best for the piles, but the caps are generally made of oak or yellow pine. The caps are generally 14 feet long (for single track) with a cross-section 12″×12″ or 12″×14″. "Split caps" would consist of two pieces 6″×12″. The sway-braces, never used for less heights than 6′, are made of 3″×12″ timber, and are spiked on with ⅜″ spikes 8″ long. The floor system will be the same as that described later for framed trestles.

134. Cost of pile trestles. The cost, per linear foot, of piling depends on the method of driving, the scarcity of suitable timber, the price of labor, the length of the piles, and the amount of shifting of the pile-driver required. The cost of soft-wood piles varies from 8 to 15 c. per lineal foot, and the cost of oak piles varies from 10 to 30 c. per foot according to the length, the longer piles costing more per foot. The cost of driving will average about $2.50 per pile, or 7.5 to 10 c. per lineal foot. Since the cost of shifting the pile-driver is quite an item in the total cost, the cost of driving a long pile would be *less* per foot than for a short pile, but on the other hand the cost of the pile is *greater* per foot, which tends to make the total cost per foot constant. Specifications generally say that the piling will be paid for per lineal foot of piling *left in the work*. The wastage of the tops of piles sawed off is always something, and is frequently very large. Sometimes a small amount per foot of piling sawed off is allowed the contractor as compensation for his loss. This reduces the contractor's risk and possibly reduces his bid by an equal or greater amount than the extra amount actually paid him.

FRAMED TRESTLES.

135. Typical design. A typical design for a framed trestle bent is given in Fig. 69. This represents, with slight variations of detail, the plan according to which a large part of the framed trestle bents of the country have been built—i.e., of those less than 20 or 30 feet in height, not requiring multiple story construction.

136. Joints. (a) **The mortise-and-tenon joint** is illustrated in

Digitized by Google

Fig. 69 and also in Fig. 66 (*a*). The tenon should be about

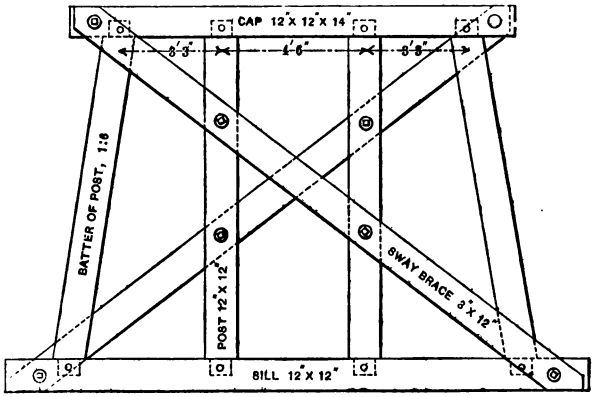


FIG. 69.

3″ thick, 8″ wide, and 5½″ long. The mortise should be cut a little deeper than the tenon. "Drip-holes" from the mortise to the outside will assist in draining off water that may accumulate in the joint and thus prevent the rapid decay that would otherwise ensue. These joints are very troublesome if a single post decays and requires renewal. It is generally required that the mortise and tenon should be thoroughly daubed with paint before putting them together. This will tend to make the joint water-tight and prevent decay from the accumulation and retention of water in the joint.

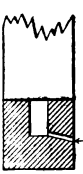


FIG. 70.

(b) **The plaster joint.** This joint is made by bolting and spiking a 3″×12″ plank on both sides of the joint. The cap and sill should be notched to receive the posts. Repairs are greatly facilitated by the use of these joints. This method has been used by the Delaware and Hudson Canal Co. [R. R.].

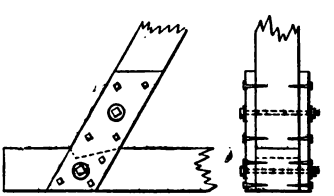


FIG. 71.

(c) **Iron plates.** An iron plate of the form shown in Fig. 72

Digitized by Google

(*b*) is bent and used as shown in Fig. 72 (*a*). Bolts passing through the bolt-holes shown secure the plates to the timbers and make a strong joint which may be readily loosened for repairs. By slight modifications in the design the method may be used for inclined posts and complicated joints.

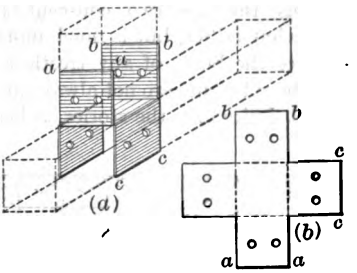


Fig. 72.

(d) **Split caps and sills.** These are described in § 129. Their advantages apply with even greater force to framed trestles.

(e) **Dowels and drift-bolts.** These joints facilitate cheap and rapid construction, but renewals and repairs are very difficult, it being almost impossible to extract a drift-bolt, which has been driven its full length, without splitting open the pieces containing it. Notwithstanding this objection they are extensively used, especially for temporary work which is not expected to be used long enough to need repairs.

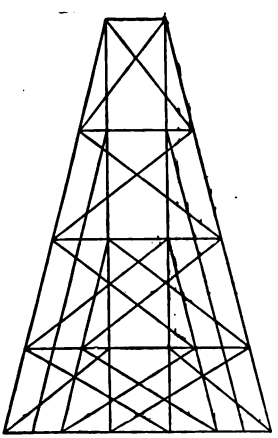


Fig. 73.

137. **Multiple-story construction.** Single-story framed trestle bents are used for heights up to 18 or 20 feet and exceptionally up to 30 feet. For greater heights some such construction as is illustrated in a skeleton design in Fig. 73 is used. By using split sills between each story and separate vertical and batter posts in each story, any piece may readily be removed and renewed if necessary. The height of these stories varies, in different designs, from 15 to 25 and even 30 feet. In some designs the structure of each story is independent of the stories above and below. This greatly facilitates both the original construction and subsequent repairs

Digitized by Google

In other designs the verticals and batter-posts are made continuous through two consecutive stories. The structure is somewhat stiffer, but is much more difficult to repair.

Since the bents of any trestle are usually of variable height and those heights are not always an even multiple of the uniform height desired for the stories, it becomes necessary to make the

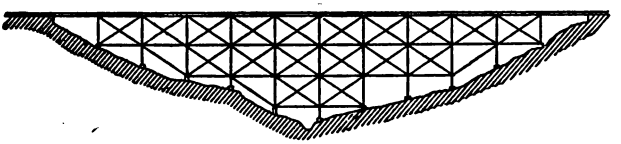


FIG. 74.

upper stories of uniform height and let the odd amount go to the lowest story, as shown in Figs. 73 and 74.

138. Span. The shorter the span the greater the number of trestle bents; the longer the span the greater the required strength of the stringers supporting the floor. Economy demands the adoption of a span that shall make the sum of these require-

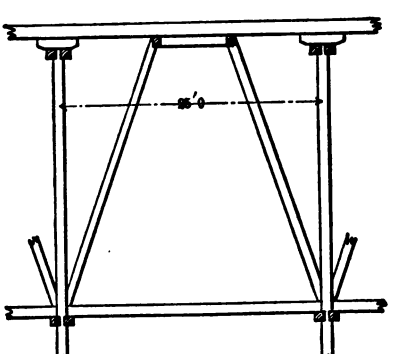


FIG. 75.

ments a minimum. The higher the trestle the greater the cost of each bent, and the greater the span that would be justifiable. Nearly all trestles have bents of variable height, but the advantage of employing uniform standard sizes is so great that many

Digitized by Google

roads use the same span and sizes of timber not only for the panels of any given trestle, but also for all trestles regardless of height. The spans generally used vary from 10 to 16 feet. The Norfolk and Western R. R. uses a span of 12′ 6″ for all single-story trestles, and a span of 25′ for all multiple-story trestles. The stringers are the same in both cases, but when the span is 25 feet, knee-braces are run from the sill of the first story below to near the middle of each set of stringers. These knee-braces are connected at the top by a "straining-beam" on which the stringers rest, thus supporting the stringer in the center and virtually reducing the span about one-half.

139. **Foundations.** (a) **Piles.** Piles are frequently used as a foundation, as in Fig. 76, particularly in soft ground, and also for temporary structures. These foundations are cheap, quickly constructed, and are particularly valuable when it is financially necessary to open the road for traffic as soon as possible and with the least expenditure of money; but there is the disadvantage of inevitable decay within a few years unless the piles are chemically treated, as will be discussed later. Chemical treatment, however, increases the cost so that such a foundation would often cost more than a foundation of stone. A pile should be driven under each post as shown in Fig. 76.

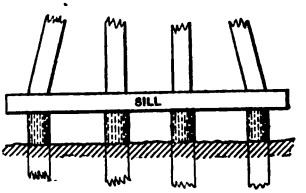


FIG. 76.

(b) **Mud-sills.** Fig. 77 illustrates the use of mud-sills as built by the Louisville and Nashville R. R. Eight blocks 12″×12″×6′ are used under each bent. When the ground is very soft, two additional timbers (12″×12″×length of bent-sill), as shown by the dotted lines, are placed underneath. The number required evidently depends on the nature of the ground.

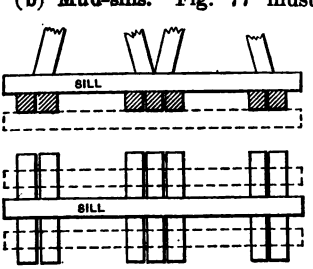


FIG. 77.

(c) **Stone foundations.** Stone foundations are the best and the most expensive. For very high trestles the Norfolk and

Digitized by Google

Western R. R. employs foundations as shown in Fig. 78, the walls being 4 feet thick. When the height of the trestle is 72 feet or less (the plans requiring for 72′ in height a foundation-wall 39′ 6″ long) the foundation is made continuous. The sill

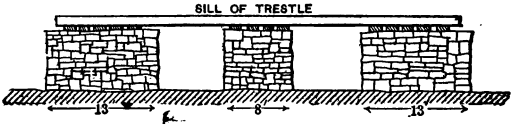


FIG. 78.

of the trestle should rest on several short lengths of 3″×12″ plank laid transverse to the sill on top of the wall.

140. **Longitudinal bracing.** This is required to give the structure longitudinal stiffness and also to reduce the columnar length of the posts. This bracing generally consists of horizontal "waling-strips" and diagonal braces. Sometimes the braces are placed wholly on the outside posts unless the trestle is very high. For single-story trestles the P. R. R. employs the "laced" system, i.e., a line of posts joining the cap of one bent with the sill of the next, and the sill of that bent with the cap of the next. Some plans employ braces forming an × in alternate panels. Connecting these braces in the center more than doubles their columnar strength. Diagonal braces, when bolted to posts, should be fastened to them as near the ends of the posts as possible. The sizes employed vary largely, depending on the clear length and on whether they are expected to act by tension or compression. 3″×12″ planks are often used when the design would require tensile strength only, and 8″×8″ posts are often used when compression may be expected.

141. **Lateral bracing.** Several of the more recent designs of trestles employ diagonal lateral bracing between the caps of adjacent bents. It adds greatly to the stiffness of the trestle and better maintains its alignment. 6″×6″ posts, forming an × and connected at the center, will answer the purpose.

142. **Abutments.** When suitable stone for masonry is at hand and a suitable subsoil for a foundation is obtainable without too much excavation, a masonry abutment will be the best. Such an abutment would probably be used when masonry footings for trestle bents were employed (§ 139, *c*).

Another method is to construct a "crib" of 10″×12″ timber,

Digitized by Google